Dynamic Network Loading of Multiple User-Classes with the Link Transmission Model

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ABSTRACT

This extended abstract shows how multiple user-classes can be considered in the link transmission model for dynamic network loading. The method assumes triangular fundamental diagrams for each user-class for which only equal shockwave speed are necessary. The link model is reconsidered; normalizations are link dependent. A simple test case is shown.

Keywords: *Dynamic Network Loading, Link Transmission Model, Multiple User-Classes*

INTRODUCTION AND LITERATURE

Dynamic Network Loading (DNL) is an important element within Dynamic Traffic Assignment. It considers the propagation of traffic flow through a road network. Generally, DNL is a large scale problem demanding high performing (simulation) models. The Link Transmission Model (LTM), see [1] and [2], is an elegant and fast model that only uses the current and past cumulative in-/outflows of the links to determine the future state. The LTM is based on Newell's simplified kinematic wave theory, see [3]. Other models (e.g. the well known Cell Transmission Model) require a space discretization, which increases the problem size tremendously. Previously proposed LTM descriptions require the First In-First Out (FIFO) assumption for the entire link and thus homogeneous vehicles. This abstract presents an extension that only requires the FIFO assumption in congested states. During free flow conditions it is possible that heterogeneous users take over. As a result, the largest drawback for applying the LTM (i.e. only one user-class is possible) is overcome by handling multiple user-classes simultaneously.

An overview of approaches for taking multiple user-classes into account can be found in [4]. The paper shows that current formulations describe the flow for each user-class as a function of the densities of all user-classes; the traffic conservation law then leads to a system of differential equations. This is typically different to our approach that is based on dedicated fundamental diagram properties and therefore does not require explicit flow formulas.

The LTM is based on properties of charcteristic lines in the time-space diagram; the challenge is to derive properties on these lines for multiple user-classes. This is achieved in this abstract by introducing so-called normalized fundamental diagrams. The resulting multiple user-class model fills a gap in the theory, it combines the fast calculations of the LTM with the ability to differentiate externabilities like emissions over heterogeneous user-classes.

The remainder of this abstract consists of the multiple user-class model description and a small test case. Due to space restictions most aspects can only be considered concisely. The full paper will as well elaborate on the performance in comparison with CTM and single class LTM, possible applications, semi congestion and model details.

MODEL DESCRIPTION

This section presents a time discretized DNL method, consisting of a link model and a node model. The road network is represented by a directed graph $(\mathcal{V}, A)$. Let T be the time horizon (i.e. the interval of interest for the study is $[0, T]$[1]). The finite set of user-classes is denoted by C. For each directed link $a \in A$ the location set X_a consists of all points on the link. The traffic flow on link $a \in A$ is fully represented by cumulative linkflows; $N_a^c(x, t)$ is the cumulative flow until time $t \in [0, T]$ on location $x \in X_a$ for user class $c \in C$. It is supposed that the several N_a^c values are normalized with respect to capacity[2]. The aggregated cumulative flow is $N_a(x,t) := \sum_{c \in C} N_a^c(x,t), \forall x \in X_a, t \in [0,T]$ The traffic flow rate $q(x,t)$ and its disaggregation in $q^c(x,t)$ for link $a \in A$ is defined as

$$q(x,t) = \frac{\partial N_a(x,t)}{\partial t} = \sum_{c \in C} \frac{\partial N_a^c(x,t)}{\partial t} = \sum_{c \in C} q^c(x,t). \tag{1}$$

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[1]Notation: $[a, b] = \{x \in \mathbb{R} | x \geq a \wedge x \leq b\}$

[2]this implies that the unit is $\neq$ *veh*, it is a fraction of the vehicle; the normalization is particularly dependent on the link and is as well user-class dependent.

The traffic density $k(x,t)$ and its disaggregation in $k^c(x,t)$ for link $a \in A$ is defined as

$$k(x,t) = -\frac{\partial N_a(x,t)}{\partial x} = -\sum_{c\in C} \frac{\partial N_a^c(x,t)}{\partial x} = \sum_{c\in C} k^c(x,t). \tag{2}$$

The link model calculates potential sending and receiving volumes using only the cumulative in- and outflow of the link. The potential receiving and sending volumes of each link are coupled in the node model.

Multiple User-Classes

The heterogeneous road users are classified into homogeneous user-classes. The traffic flow characteristics of a user-class $c \in C$ are considered to be constant on a dedicated directed link and are depicted with a k^c—q^c fundamental diagram. It is supposed that the fundamental diagram has Newell's triangular form. The fundamental diagrams of a user-class represents the corresponding homogeneous case (i.e. a dedicated road). For each link (type) and each user-class a fundamental diagram is defined. There is one user-class independent property, namely the shockwave speed ω_a; the speed should be equal for a specific link (type). The characteristics of each user-class $c \in C$ on each link $a \in A$ are then fully represented by two of the following variables: (1) free-flow speed γ_a^c $\left(\text{in } \frac{km}{h}\right)$, (2) jam density $\tilde{K}_a^c$ $\left(\text{in } \frac{veh}{km}\right)$ and (3) capacity $\tilde{Q}_a^c$ $\left(\text{in } \frac{veh}{h}\right)$. The tildes on $\tilde{K}_a^c$ and $\tilde{Q}_a^c$ indicate non-normalized variables. Speed is invariant under normalization. By specifying two variables, the third variable can be determined using the others in combination with the shockwave speed.

The use of a normalization is already mentioned; it is necessary to build a sound mathematical model. The normalization is done with respect to the capacity; it is performed for each link $a \in A$ and each user-class $c \in C$ and comprehends dividing by $\tilde{Q}_a^c$.[3] The unit of the model becomes *normalized vehicle* (denoted as *nv*) instead of vehicle. The resulting jam density K_a^c $\left(\text{in } \frac{nv}{km}\right)$ becomes $K_a^c = \tilde{K}_a^c/\tilde{Q}_a^c$ and the capacity $Q_a^c = 1$ $\left(\text{in } \frac{km}{h}\right)$. Figure 1 shows the fundamental diagrams for two user-classes, figure 2 shows the normalized diagrams. The term passenger car equivalent is avoided since it is usually defined as the ratio between road occupancy of different vehicle types.

Link model

The constraints and equations derived in this paragraph are valid for all $t \in [0,T]$. Let $a \in A$ be a link with length L_a, jam density $\tilde{K}_a^c$, shockwave speed ω_a, free flow speeds γ_a^c and capacities $\tilde{Q}_a^c$. Let x_a be the entry point of link a, then define $U_a^c(t) := N(x_a,t)$ as the cumulative inflow of link a until time t and define $V_a^c(t) := N(x_a+L_a,t)$ as the cumulative outflow of link a until time t. The link model results in the (to be defined) potential sending volumes and a potential receiving volume which represent an upper bound on the throughput of the link in a certain interval.

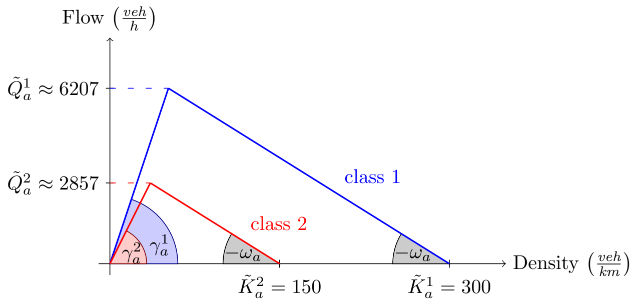

Fig. 1: k^c—q^c fundamental diagram for two user-classes

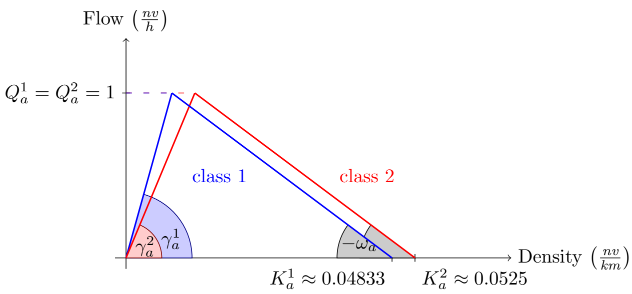

Fig. 2: Normalized k^c—q^c fundamental diagram for two user-classes

During free-flow conditions all vehicles can tranverse the link with their free-flow speed. In this case the state of a user-class is independent of the other user-classes and it is constant over vehicle trajectories. A user-class c vehicle entering the link at time t will — in free-flow state — exit the link at time $t+L_a/\gamma_a^c$. The (constant) traffic state is denoted by q^c and k^c. If a generalization of the fundamental theorem of calculus[4] is applied, it follows that

$$\begin{aligned} & N^c\left(x_a + L_a, t + \frac{L_a}{\gamma_a^c}\right) - N^c(x_a,t) \\ =& \int_l \nabla N^c(x,t)ds = \int_l q^c(x,t)dt - k^c(x,t)dx \\ =& q^c\frac{L_a}{\gamma_a^c} - k^c L_a = L_a\left(\frac{q^c}{\gamma_a^c} - k^c\right) = 0, \end{aligned} \tag{3}$$

where l can be any piecewise C^1 path[5] $l : \left[(x_a,t), \left(x_a+L_a, t+\frac{L_a}{\gamma_a^c}\right)\right] \to \mathbb{R}^2$, take the vehicle trajectory in this case. Note that the user-class specific

[3] i.e. in the normalized model the capacity is always equal to 1.

[4] The literature states that this is an application of Green's theorem, this is incorrect. Green's theorem describes the relation between integrals over regions and its boundaries.

[5] C^1 is the class of continuously differentiable functions.

cumulative flow $N_a^c(x,t)$ is constant over each trajectory (this is the result of (3)) and that it is possible for the faster class to take over the slower class.

During congested conditions shockwaves originate. Shockwaves traverse from the end of the link to the direction of the beginning of the link with the shockwave speed ω_a. The traffic state is constant over the shockwave trajectory (i.e. the upstream path with velocity ω_a) and is in the single class LTM characterized by q and k. Since the composition of classes is variant over the shockwave, q and k can differ over the characteristic line. Supposed that a shockwave, originated at time t, does not dissolve, then it will reach the beginning of the link at time $t - L_a/\omega_a$. If a generalization of the fundamental theorem of calculus is applied it follows that

$$\begin{aligned}
&N\left(x_a,t\right) - N\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right) \\
&= \int_l \nabla N(x,t)ds = \int_l q(x,t)dt - k(x,t)dx \\
&= \lim_{n\to\infty} \sum_{i_n=1}^{N_n} q_{i_n}\left(t_{i_n} - t_{i_n-1}\right) - k_{i_n}\left(x_{i_n} - x_{i_n-1}\right) \\
&= \lim_{n\to\infty} \sum_{i_n=1}^{N_n} q_{i_n}\left(-\frac{x_{i_n} - x_{i_n-1}}{\omega_a}\right) - k_{i_n}\left(x_{i_n} - x_{i_n-1}\right) \\
&= \lim_{n\to\infty} \sum_{i_n=1}^{N_n} \left(x_{i_n} - x_{i_n-1}\right)\left(\frac{-q_{i_n}}{\omega_a} - k_{i_n}\right) \\
&= \lim_{n\to\infty} \sum_{i_n=1}^{N_n} \left(x_{i_n} - x_{i_n-1}\right)\left(K_{i_n} + k_{i_n} - k_{i_n}\right) \\
&= \lim_{n\to\infty} \sum_{i_n=1}^{N_n} \left(x_{i_n} - x_{i_n-1}\right) \\
&\quad \cdot \sum_{c\in C} \frac{N^c\left(x_{i_n-1}, t_{i_n-1}\right) - N^c\left(x_{i_n}, t_{i_n}\right)}{N\left(x_{i_n-1}, t_{i_n-1}\right) - N\left(x_{i_n}, t_{i_n}\right)} K^c \\
&= L_a \sum_{c\in C} \frac{N^c\left(x_a,t\right) - N^c\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right)}{N\left(x_a,t\right) - N\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right)} K^c
\end{aligned} \tag{4}$$

where l can be any piecewise C^1 path $l : \left[(x_a + L_a, t), \left(x_a, t - \frac{L_a}{\omega_a}\right)\right] \to \mathbb{R}^2$ (take the shockwave trajectory in this case), where $\left\{\{x_{i_n}\}_{i_n=0}^{N_n}, \{y_{i_n}\}_{i_n=0}^{N_n}\right\}_{n=1}^{\infty}$ is the series of partitions of $\left[(x_a + L_a, t), \left(x_a, t - \frac{L_a}{\omega_a}\right)\right]$ (i.e. l) corresponding to the lower Riemann definition, where $q_{i_n} = q(x_{i_n-1}, t_{i_n-1}), k_{i_n} = k(x_{i_n-1}, t_{i_n-1})$ represent the traffic state on the line $[(x_{i_n-1}, t_{i_n-1}), (x_{i_n}, t_{i_n})]$ which has jam density K_{i_n}[6]. From (4) it follows that a shockwave traversing from the end to the beginning of a link encounters $L_a \sum_{c\in C} \frac{N^c(x_a,t) - N^c\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right)}{N(x_a,t) - N\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right)} K^c$ nvs.

[6] which is equal to $\sum_{c\in C} \frac{N^c\left(x_{i_n-1}, t_{i_n-1}\right) - N^c\left(x_{i_n}, t_{i_n}\right)}{N\left(x_{i_n-1}, t_{i_n-1}\right) - N\left(x_{i_n}, t_{i_n}\right)} K^c$

Potential sending volume

Definition 1 (Potential sending volume)
Let $a \in A$, $c \in C$, $t \in [0,T)$ and $\Delta t \in (0, T-t]$, then the *potential sending volume* $S_a^c(t, \Delta t)$ is the traffic quantity of user-class c that can exit link a during time interval $[t, t+\Delta t]$ assuming an infinite downstream capacity and respecting given previous states upto time t (i.e. $U^c(t')$ and $V^c(t')$ are given for $t' \in [0,t]$).

The potential sending volume is equal to the increase of the cumulative flow at the end of the link during the interval. Furthermore it is bounded from above if free-flow state is considered, in that case (3) can be applied. This formally leads to

$$\begin{aligned}
S_a^c(t, \Delta t) &= N^c(x_a + L_a, t + \Delta t) - N^c(x_a + L_a, t) \\
&\le N^c\left(x_a, t + \Delta t - \frac{L_a}{\gamma_a^c}\right) - V_a^c(t) \\
&= U_a^c\left(t + \Delta t - \frac{L_a}{\gamma_a^c}\right) - V_a^c(t) =: \bar{S}_a^c(t, \Delta t)
\end{aligned} \tag{5}$$

which states that the potential sending volume cannot be larger than the number of nv that entered — but did not exit (before time t) — the link and is present (on time $t + \Delta t$) at least than the time needed to traverse the link with free-flow speed.

If $\sum_{c\in C} \bar{S}_a^c(t, \Delta t) > Q_a \Delta t = \Delta t$ then it exceeds the link capacity and therefore the total potential outflow should be equal to the capacity then. An algorithm is developed to determine the mixture of classes at $x_a + L_a$ at time t on which will be elaborated in the full paper. It results in sending flows that comply with $\sum_{c\in C} S_a^c(t, \Delta t) \le \Delta t$.

Potential receiving volume

Definition 2 (Potential receiving volume)
Let $a \in A$, $t \in [0,T]$ and $\Delta t \in [0, T-t]$, then the *potential receiving volume* $R_a(t, \Delta t)$ is the traffic quantity that can enter link a during time interval $[t, t+\Delta t]$ assuming an infinite upstream flow and respecting given previous states upto time t (i.e. $U^c(t')$ and $V^c(t')$ are given for $t' \in [0,t]$).

Denote that — contrary to the potential sending volume, which is defined per user-class — the potential receiving volume is defined aggregated over all user-classes. The potential receiving volume is the increase of the cumulative flow at the start of the link during the interval. If we suppose the link is in a congested state, then (4) constitutes a bound from above. This formally leads to

$$
\begin{aligned}
R(t,\Delta t) &= N(x_a, t+\Delta t) - N(x_a, t) \\
&\leq N\left(x_a + L_a, t+\Delta t - \frac{L_a}{\omega_a}\right) - N(x_a, t) \\
&+ L_a \sum_{c\in C} \frac{N^c(x_a,t) - N^c\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right)}{N(x_a,t) - N\left(x_a + L_a, t - \frac{L_a}{\omega_a}\right)} K^c \\
&= V\left(t + \Delta t - \frac{L_a}{\omega_a}\right) - U(t) \\
&+ L_a \sum_{c\in C} \frac{U^c(t) - V^c\left(t - \frac{L_a}{\omega_a}\right)}{U(t) - V\left(t - \frac{L_a}{\omega_a}\right)} K^c =: \bar{R}_a(t,\Delta t)
\end{aligned}
\quad (6)
$$

which states that the potential receiving volume cannot be larger than the number of *nv*s that is not 'caught' by the shockwave arriving (at x_a) at time $t+\Delta$.

The link capacity gives that $R(t,\Delta t) \leq \Delta t$, together with (6) and the fact that one bound must be met leads to $R(t,\Delta t) = \min\left\{\Delta t, \bar{R}_a(t,\Delta t)\right\}$.

Node model

In the node model the normalized potential sending and receiving volumes are converted to vehicles (i.e. in *veh*). At each node the different potential volumes are connected and lead to the final increase of cumulative flows in each interval. Known capacity based node models can be used. There will be more elaboration on the node model in the full paper.

TEST CASE

For the test case with two user-classes a bottleneck is created with two links. The first has 3 lanes, the second has 2 lanes. Maximum speed of the two classes is respectively 120 km/h (cars) and 80 km/h (trucks). Figures 1 and 2 show the fundamental diagrams for the 3 lane link ($\omega = 25$). A 20 minute simulation is performed, where the demand was $(4000, 500)veh/h$ in the first 5 minutes, $(500, 2500)veh/h$ in the second 5 minutes and $(500, 500)veh/h$ in the last 10 minutes. So congestion should originate on link 1 and dissolve later on.

Figure 3 shows the cumulative flows at beginning of link 1 (U), at the node (M) and at the end of link 2 (V). Travel times for link 1 are shown in figure 4. The figures show the different travel behaviour for classes, justifying the model. Some observations: (1) the first cars that overtook all trucks do not encounter congestion, (2) there is a short period of decrease in travel times since the trucks departing just after 5 minutes will arrive later at the beginning of the queue than the cars departing just before 5 minutes and (3) different outflow mixtures are observed; the outflow mixture changes to 50-50 in the end.

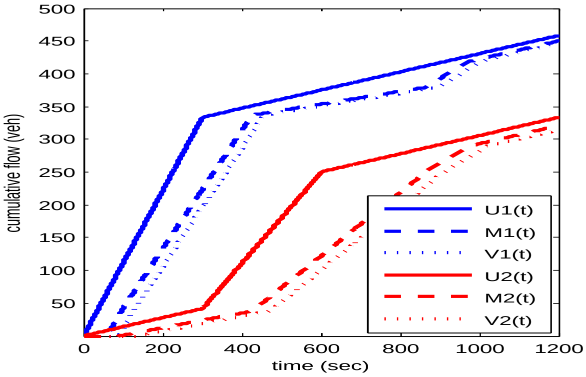

Fig. 3: Cumulative in-, mid- and outflow for two classes

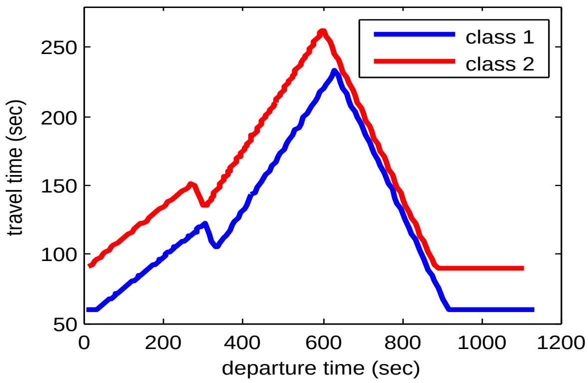

Fig. 4: Travel times for two classes

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