Simultaneous Search for Differentiated Products: The Impact of Search Costs and Firm Prominence

Short title: "Simultaneous Search for Differentiated Goods"*†

José L. Moraga-González, Zsolt Sándor and Matthijs R. Wildenbeest

May 2020

Abstract

We extend the literature on simultaneous search by allowing for differentiated products and search cost heterogeneity. We show conditions under which a symmetric price equilibrium exists. We provide a necessary and sufficient condition under which an increase search costs may result in a lower, equal, or higher equilibrium price. We extend this analysis to the case with more than two firms. The effects of prominence on equilibrium prices are also studied. The prominent firm charges a higher price than the non-prominent firm and both their prices are below the symmetric equilibrium price. Consequently, market prominence increases the consumers' surplus.

Keywords: non-sequential search, simultaneous search, oligopoly, search cost heterogeneity, differentiated products, non-uniform sampling, prominence

JEL Classification: D43, C72

---

*Corresponding author: José L. Moraga-González, Department of Economics, School of Business and Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. E-mail: `j.l.moragagonzalez@vu.nl`

†We have benefited from the comments of an Editor and two anonymous referees, as well as from presentations at Universidad Carlos III Madrid and CEMFI. The discussants and the audiences at the 4th Workshop on Search and Switching Costs (Higher School of Economics, Moscow, May 2013), the EARIE 2018 (Athens) and the MaCCI Annual Conference 2020 (Mannheim) are also gratefully acknowledged. Sándor thanks financial support from grant PN-II-ID-PCE-2012-4-0066 of the Romanian Ministry of National Education, CNCS-UEFISCDI. Parts of this paper were included in an earlier manuscript entitled "Do higher search costs make markets less competitive?"

1 Introduction

The early consumer search literature, which dates back at least to the 1960s, was dominated by homogeneous product models and focused on how search costs limited consumer price discovery, which often resulted in price dispersion (Stigler, 1961; Burdett and Judd, 1983; Stahl, 1989). With the rise of the Internet, it has become evident that search frictions, by constraining not only price but also product choice sets, distort consumer choice further. To properly capture this important feature, the more recent consumer search literature has focused on modelling markets for differentiated products. Moreover, following Weitzman (1979) and Wolinsky (1983; 1986), the accent has been put on models of *sequential* consumer search (Anderson and Renault, 1999; Armstrong *et al.*, 2009; Moraga-González and Petrikaitė, 2013).

This emphasis on sequential consumer search is not always justified because, depending on the context, *simultaneous* search, also referred to as non-sequential or fixed-sample-size search, may be superior to sequential search (Morgan and Manning, 1985). Further, empirically it seems that in some industries simultaneous search is more prevalent than sequential search. For example, recent work by De Los Santos *et al.* (2012) and Honka and Chintagunta (2017) has shown that for books and car insurance sold online, observed consumer search patterns are consistent with simultaneous search.[1] Furthermore, because search decisions do not depend on search outcomes when searching simultaneously, obtaining closed-form expressions for purchase probabilities and market shares is

[1]To be able to empirically distinguish between sequential and simultaneous search, De Los Santos *et al.* (2012) exploit data on the sequence of searches and focus on a crucial difference between the two search methods in terms how search outcomes affect search behaviour: when consumers search sequentially, the decision to continue searching depends on the outcome of the search, while with simultaneous search consumers commit to a certain number of searches before seeing any search outcomes. Honka and Chintagunta (2017) propose a test that only requires data on consideration sets.

relatively easy and this has made models of simultaneous search for differentiated products popular in recent empirical work (see, e.g., De Los Santos *et al.*, 2012; Honka, 2014; Moraga-González *et al.* 2015; Pires, 2016, 2018; Ershov, 2018; Murry and Zhou, 2019; Lin and Wildenbeest, 2019; and Donna *et al.* 2019).

Despite this, market models of simultaneous search for differentiated products remain understudied in the theoretical literature. The purpose of this paper is narrowing this gap by extending the literature on consumer search for differentiated products to allow for *simultaneous* consumer search and consumer search cost heterogeneity. Within this framework, we derive novel results concerning the impact of search costs on competition and the effect of firm prominence on prices.

To the best of our knowledge, Anderson *et al.* (1992, p. 246) is the only theoretical study of equilibrium pricing with simultaneous consumer search for differentiated products.[2] In their model, N firms offer differentiated products to consumers who initially do not know how much the products are worth to them. The value of the match between a consumer and a product is a random draw from the type I extreme value distribution. Only after paying a search cost, a consumer can learn the value she places on a given product. Firms are symmetric and all consumers have the same utility and search cost. The problem of a consumer is thus choosing how many products to inspect; after having learned the match values of the inspected products, the consumer picks the product that yields the highest utility.

In contrast to markets for homogeneous products in which consumers turn out to optimally choose to sample the prices of at most two firms (Burdett and Judd, 1983; Janssen and Moraga-González, 2004), Anderson *et al.* (1992) show that with differentiated products, depending on the magnitude

[2]For an authoritative and up-to-date survey of firm pricing with consumer search, see Anderson and Renault (2018).

of the search cost, consumers may check the products of any number of firms (including all of them if the search cost is sufficiently low). Specifically, they show that the equilibrium price in the search model is equal to the Perloff and Salop (1985) (full information) price that would prevail in a market where the number of competitors is equal to the sample size selected by consumers. In equilibrium the price is therefore insensitive to the number of sellers. Furthermore, small increases in the search cost do not affect the equilibrium price; it is only when the search cost increases by a sufficiently large amount that consumers choose to inspect fewer products, which results in a higher equilibrium price. Finally, market settings in which some firms are more salient than others (as in the prominence model of Armstrong *et al.*, 2009) are no different from symmetric market environments.

These three rigidities, namely that prices respond neither to small changes in search costs, nor to variations in the number of competitors, nor to differences in the market saliency of the firms, are somewhat unsatisfactory model features and we believe they are responsible for the fact that simultaneous search has received less attention in the theoretical literature on search for differentiated products than in the empirical literature. To deal with these limitations, in Section 2 we introduce a new model that allows for search cost heterogeneity. When consumers differ in their costs of search, they optimally choose to inspect different numbers of products. Specifically, consumers with sufficiently low search costs choose to check all available products; consumers with higher search costs choose to inspect a subset of the products, the higher their search costs the smaller the subset of products they inspect; and consumers with prohibitively high search costs do not search at all and drop out of the market altogether. A consumer search equilibrium is then a partition of the consumer population into subsets of consumers inspecting different numbers of products. From the point of view of an individual firm, consumers who check many products are more price sensitive than consumers who inspect just a few. Optimal pricing makes a tradeoff between the incentives to extract profits from the less price sensitive consumers and the incentives to compete for the more price sensitive ones. As we vary the number of firms, or change the search cost distribution, the partition resulting from consumer equilibrium behaviour changes smoothly, which also smoothly changes the equilibrium price. Furthermore, as sampling becomes less uniform due to a firm's enhanced market prominence, the allocation of consumers to firms changes continuously, which is also reflected in the price equilibrium.

In Section 3 of the paper we use the new model to derive the following results. We first study the case of duopoly and present the characterisation of a symmetric pure-strategy price equilibrium. For any arbitrary search cost distribution, we show that an equilibrium exists if the distribution of match values is uniform. If the search cost distribution is also uniform, then the equilibrium is unique. More general results are hard to obtain because the demand of a firm consists of a weighted sum of the demand arising from the consumers who choose to inspect one product only and the demand stemming from the consumers who choose to inspect the products of the two firms. Even though the profit contributions arising from each of these demands can be shown to be quasi-concave, the sum might not be well behaved. Nevertheless, we show that a symmetric pure-strategy price equilibrium also exists when the distribution of match values is quadratic and convex and the distribution of search costs is quadratic and concave.

We then proceed in Section 4 with an examination of how the equilibrium price responds to increases in search costs in the duopoly model. We extend insights from Moraga-González *et al.*

(2017a) to the case of simultaneous search for differentiated products and provide conditions under which the equilibrium price increases, remains constant, or decreases as search costs rise. These conditions involve the impact of an increase in the costs of search on two margins of search, namely, the intensive search margin (or search intensity) and the extensive search margin (or the decision to search at all). Regarding the extensive search margin, an increase in search costs tends to increase the elasticity of demand because high-search-cost consumers drop out of the market altogether whereas regarding the intensive search margin, an increase in search costs tends to decrease the elasticity of demand because consumers search less. Which of these two effects dominates depends on the properties of the search cost distribution.

We identify a necessary and sufficient condition under which higher search costs for all consumers result in a lower equilibrium price. This necessary and sufficient condition is quite distinct from the condition that would arise under sequential search (as in Moraga-González *et al.* 2017a). In fact, it may happen that a change in search costs will have the opposite effect on equilibrium prices in the sequential search model than in the simultaneous search model. This observation has a major implication for the empirical researcher interested in the understanding of the impact of a reduction in search costs. Even if a mis-specification of the search protocol does not bias the estimation of the search cost distribution, counterfactual analysis of lower search costs may lead to wrong conclusions.

Finally, we identify a stochastic ordering of distributions, called the *reversed hazard rate* ordering, such that higher search costs result in lower (higher) prices when the search cost distribution exhibits the decreasing (increasing) reversed hazard rate property. The decreasing (increasing) reversed hazard rate property is equivalent to the notion of log-submodularity (log-supermodularity) of the cumulative distribution function. Intuitively, when the search cost distribution is log-submodular (log-supermodular) an increase in search costs is more (less) noticeable at lower than at higher quantiles, which implies that the share of consumers inspecting the two products relative to the share of consumers inspecting just one increases (decreases) and the equilibrium price correspondingly goes down (up). Whether a search cost distribution is log-supermodular or log-submodular is empirically testable and the outcome of such a test will be useful when predicting the effects of policies that improve search technologies or increase market transparency.

In Section 5 we extend our results in two directions. In Section 5.1 we consider the case of $N > 2$ firms, provide the characterisation of the price equilibrium and, drawing from a recent contribution by Choi and Smith (2017) about preservation of quasi-concavity under aggregation, give conditions for the existence of equilibrium. We show that, for any search cost distribution, an equilibrium exists in markets with fewer than nine firms when the distribution of match values is uniform. With a larger number of firms, the existence of equilibrium is guaranteed provided that the marginal cost of production is sufficiently high.

In Section 5.2 we return to the duopoly model and examine the case in which the firms differ in the likelihood with which they are sampled by consumers (Hortaçsu and Syverson, 2004; De los Santos, 2018). Intuitively, non-uniform sampling creates a market asymmetry in favour of the salient firm because the consumers who visit it have higher search costs on average than the consumers who visit the non-salient firm. As a result, the salient firm charges a higher price and obtains higher profits than the non-salient one. Our result is consistent with McDevitt (2014), who finds that plumbing firms in Chicago with a name that begins with an A or a number, and are therefore more likely to be searched first when using the Yellow Pages, command a price premium that is 8.4 percent above the average.

Interestingly, in contrast to the study of prominence of Armstrong *et al.* (2009), in which consumers search sequentially, in our model with simultaneous search market saliency does not hurt consumers. In fact, when one firm is prominent and is therefore visited by all the consumers who choose to inspect only one product, in the unique equilibrium both the prominent and the non-prominent firms charge lower prices and consumer surplus is thus higher than when the firms are equally likely to be visited by consumers. Numerical results confirm this insight for less extreme situations of saliency.

1.1 Related literature

The literature on consumer search can be classified in terms of the search protocol and whether or not products are horizontally differentiated. Most of the early papers are about homogeneous product markets. A key contribution is Diamond (1971), who demonstrated that when consumers search sequentially to discover lower prices for a homogenous product, the unique pricing equilibrium is the monopoly price. Stahl (1989) introduced a simple form of search cost heterogeneity into Diamond's framework (the well-known and much-used 'shoppers and non-shoppers' formulation) and derived an equilibrium with price dispersion. Dealing with more general forms of consumer search cost heterogeneity in models of sequential consumer search with homogeneous product sellers has proven to be quite difficult (Stahl, 1996).

Burdett and Judd (1983) used a model of simultaneous search to show that an equilibrium with price dispersion also exists in the absence of search cost heterogeneity. Janssen and Moraga-González

(2004) extended the setting of Burdett and Judd to oligopoly and allowed for an atom of shoppers. Their main results are on the effects of entry. Hong and Shum (2006) were the first to introduce general forms of consumer search cost heterogeneity in Burdett and Judd's framework.[3] However, they did this for the purpose of estimation and did not provide existence of equilibrium or comparative statics results. Moraga-González *et al.* (2017b) prove the existence of a mixed strategy equilibrium in such a model and present new results on the relationship between prices and the number of firms. In the online Appendix of Moraga-González *et al.* (2017a), an analysis of how search costs affect prices in such a model is provided.

Weitzman (1979) is the first paper that studies optimal consumer search for differentiated products. Wolinsky (1983; 1986) are early papers embedding sequential consumer search for differentiated products into market settings. These papers show that, because of product differentiation, monopoly pricing is not an equilibrium. Hence, product differentiation invalidates the Diamond paradox. Moreover, with infinitely many firms, because consumers have positive search costs, prices remain above the marginal cost (Wolinsky, 1986). Anderson and Renault (1999) developed the model further and proved that prices increase when search costs rise, the number of firms decreases or products become less differentiated, and that entry is excessive from a welfare perspective.

Wolinsky's model is nowadays regarded as the workhorse model of sequential search for differentiated products in the consumer search literature. As such, it has seen numerous extensions in recent years. One such extension is the study of prominence of Armstrong *et al.* (2009) mentioned earlier (see also Wilson, 2010; Rhodes, 2011; Zhou, 2011; and Fishman and Lubensky, 2018). Re-

[3] See also Moraga-González and Wildenbeest (2008), Wildenbeest (2011), Moraga-González *et al.* (2013), and Sanches *et al.* (2018).

latedly, Armstrong and Zhou (2011) and Haan and Moraga-González (2011) present models in which a seller's market prominence depends on its choice of strategic variables such as price or advertising intensity.

In another related paper, Moraga-González *et al.* (2017a) extend Wolinsky's model by allowing for arbitrary search cost densities. They provide conditions for existence and uniqueness of equilibrium and derive the comparative statics effects of higher search costs. Like in this paper, they find that prices can increase or decrease when search costs go up provided that some consumers choose to not search in equilibrium. However, because they deal with sequential search, their sufficient conditions for prices to increase or decrease in search costs are based on properties of search cost densities (specifically, the likelihood ratio ordering), rather than of search cost distributions (reversed hazard rate ordering), which are weaker and can be applied more generally.

2 Model

In this section we present a *duopoly* model of firms selling horizontally differentiated products to consumers who search the market for satisfactory goods using a simultaneous search strategy.[4] The two firms produce the horizontally differentiated products at a marginal cost equal to r and choose their prices simultaneously to maximise profits. We focus on pure-strategy symmetric Nash equilibria (SNE); let p^* denote a SNE price.

There is a unit mass of consumers. A consumer m has tastes for a product i described by the

[4]The N-firm model is presented in Section 5.1, where we show that the main insights of the duopoly model carry over to the more general oligopoly case.

following indirect utility function:

$$u_{im} = \begin{cases} \varepsilon_{im} - p_i & \text{if she buys product } i \text{ at price } p_i; \\ 0 & \text{otherwise.} \end{cases}$$

The parameter ε_{im} is a match value between consumer m and product i. The match value ε_{im} is assumed to be i.i.d. across consumers and products. Let F be the cumulative distribution function of ε_{im}, defined over the support $[0, \overline{\varepsilon}]$. We assume that the density function of match values, denoted f, is differentiable and log-concave.

Consumers search simultaneously for a satisfactory product. This means that they first choose the number of firms to visit, including possibly none, in order to maximise expected utility. Once they have visited the desired number of firms, they buy from the store offering them the best deal, or else they do not buy anything. While deciding on the intensity of search, they hold correct conjectures about the equilibrium price. The total cost of search of a consumer with search cost c_m who searches $n = 0, 1, 2$ times is nc_m. Consumers have heterogeneous search costs. The distribution of search costs is denoted G and the density g; we assume that g is positive on the support $(\underline{c}, \overline{c})$. The lower bound $\underline{c}$ does not play much of a role so we will set it equal to 0 in most of what follows. The upper bound $\overline{c}$ does play a very important role in the analysis that follows because it drives consumer search participation. When $\overline{c}$ is large enough, the market is not covered in the sense that not all consumers choose to search. Though not the most tractable, this case is the most interesting when analysing the impact of higher search costs and will therefore be the focus of our analysis in Sections 3 and 4.[5]

[5]When $\overline{c}$ is low enough, all consumers choose to check the products of the two firms and the situation is thus similar to that in which consumers have perfect information. For intermediate levels of $\overline{c}$ all consumers search but some check the product of one firm only and the rest check both products. The latter two cases are discussed in the working paper version of this article (Moraga-González *et al.*, 2020).

To put our model in perspective, it is a duopoly version of the workhorse search model of Wolinsky (1986), but with search cost heterogeneity and simultaneous search instead of sequential search. Later in the paper we also discuss the N-firm case. The critical distinction between sequential and simultaneous search is that with simultaneous search consumers commit ex-ante to a number of searches. As mentioned in the introduction, only Anderson, De Palma, and Thisse (1992) have theoretically analysed simultaneous search for differentiated products. In their model, all consumers have the same search cost and this results in an equilibrium where all of them inspect the same number of products. With arbitrary search cost heterogeneity, different consumers pursue distinct search strategies including the possibility of not searching at all.

3 Equilibrium

In this section we characterise a pure-strategy symmetric Nash equilibrium. We begin by examining the problem of the consumers. Assume both firms charge a price $p^* \in [r, p^m]$, where p^m denotes the standard monopoly price.[6] Because consumers have correct expectations about the equilibrium price, a consumer with search cost c that chooses to only inspect the product of one firm expects to obtain a utility equal to:

$$U(1,c) = \Pr[\varepsilon \geq p^*] \left(E[\varepsilon|\varepsilon \geq p^*] - p^*\right) - c = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c, \tag{1}$$

where $\Pr[\varepsilon \geq p^*]$ is the probability that ε is at least p^* and E is the expectation operator. If the consumer instead chooses to inspect the products of the two firms, the utility she expects to obtain

[6] That is, $p^m = \arg\max_p (p-r)(1-F(p))$.

is equal to:

$$U(2,c) = \Pr[z_2 \geq p^*]\left(E[z_2|z_2 \geq p^*] - p^*\right) - 2c = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)2F(\varepsilon)f(\varepsilon)d\varepsilon - 2c, \tag{2}$$

where $z_2 \equiv \max\{\varepsilon_1, \varepsilon_2\}$ and has a cumulative distribution function equal to $F(\varepsilon)^2$.

For a consumer to conduct at least one search, $U(1,c)$ has to be positive. Correspondingly, let us define the critical search cost value:

$$c_0(p^*) \equiv \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)f(\varepsilon)d\varepsilon. \tag{3}$$

Consumers for whom $c > c_0(p^*)$ do not find it worthwhile to conduct any search. We denote by

$$\mu_0(p^*) \equiv 1 - G(c_0(p^*))$$

the total mass of consumers who abstain from searching in this market.

For a consumer to check the products of the two firms rather than the product of only one firm, $U(2,c)$ has to be greater than $U(1,c)$. Correspondingly, we define the critical search cost value:

$$c_1(p^*) \equiv \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1]f(\varepsilon)d\varepsilon. \tag{4}$$

Consumers for whom $c < c_1(p^*)$ prefer to check two products while consumers for whom $c_1(p^*) < c < c_0(p^*)$ prefer to check only one. Denoting the mass of consumers searching k times by $\mu_k(p^*)$, we have:

$$\mu_1(p^*) = G(c_0(p^*)) - G(c_1(p^*)); \text{ and } \mu_2(p^*) = G(c_1(p^*)). \tag{5}$$

Our assumptions on the search cost distribution imply that $\mu_1(p^*) + \mu_2(p^*) < 1$ and $\mu_k(p^*) > 0$, $k = 0, 1$.

Figure 1 illustrates how the consumer population is partitioned into consumers that leave the market altogether, consumers that check one product only, and consumers who check both products.

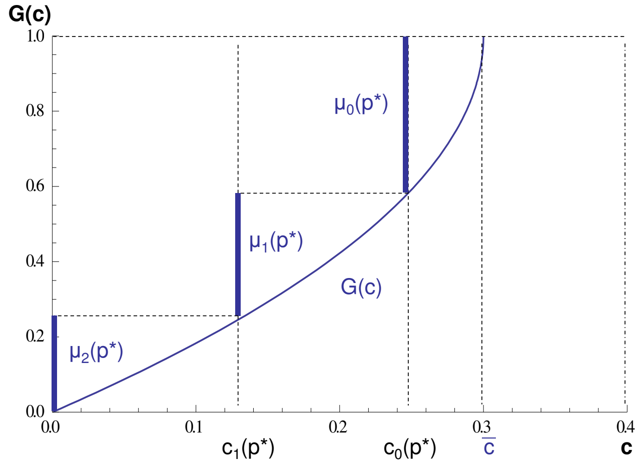

Figure 1: *Equilibrium Search Intensities And The Search Cost CDF*

We now move to the problem of the firms. To characterise the symmetric pure-strategy equilibrium we start by deriving the payoff of a firm i that deviates from equilibrium pricing by charging a price $p_i \neq p^*$, given that the rival firm charges p^*, and given consumer search behaviour. The expected payoff to the deviant firm i is:

$$\pi_i(p_i; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}\Pr[\varepsilon_i \geq p_i] + \mu_2(p^*)\Pr\left[\varepsilon_i - p_i \geq \max\{\varepsilon_j - p^*, 0\}\right]\right). \tag{6}$$

This payoff formula is easily understood. The per-consumer profit is $p_i - r$. Consumers that check only one product will pick firm i's good with probability 1/2; these consumers buy firm i's product when the match values they obtain there are higher than the price p_i. Consumers that check the two products only buy from firm i when firm i's deal is better than the rival's and the outside option of 0. Thus, the payoff can be seen as a weighted average of the perfect information monopoly payoff and the duopoly payoff, though the weights, which depend on the search cost distribution, do not sum up to 1.

When firm i deviates by charging a higher price than the rival, i.e. $p_i > p^*$, the payoff in equation (6) can be written as follows:[7]

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - (p_i - p^*))f(\varepsilon)d\varepsilon\right). \tag{8}$$

Taking the first order condition, setting $p_i = p^*$, and replacing $\mu_1(p^*)$ and $\mu_2(p^*)$ by their corresponding values, we obtain the necessary condition for a SNE price p^*:[8]

$$H\left(p^*\right) = 0, \tag{9}$$

where

$$H\left(p\right) \equiv N(p)G\left(c_1(p)\right) - D(p)G\left(c_0(p)\right), \tag{10}$$

and $D(p)$ and $N(p)$ are given by

$$D(p) \equiv -\left[1 - F(p) - (p - r)\, f(p)\right];$$

$$N(p) \equiv F(p)(1 - F(p)) - 2\,(p - r)\left(\int_{p}^{\overline{\varepsilon}} f\left(\varepsilon\right)^2 d\varepsilon + F(p)f(p) - \frac{1}{2}f(p)\right).$$

Even though equation (9) cannot be solved explicitly for p^*, we show in the Appendix that a candidate equilibrium price $p^* \in [r, p^m]$ exists for any F and G. Furthermore, we obtain the following result:

[7] When firm i deviates by charging a lower price than the rival, i.e. $p_i < p^*$, the payoff formula is different:

$$\pi_i(p_i < p^*; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\left[1 - F(\overline{\varepsilon} + p_i - p^*) + \int_{p_i}^{\overline{\varepsilon}+p_i-p^*} F(\varepsilon - (p_i - p^*))f(\varepsilon)d\varepsilon\right]\right). \tag{7}$$

However, the condition that a symmetric price equilibrium must satisfy is the same as the one in equation (9).

[8] For details on this and later derivations, see the working paper version of this article.

PROPOSITION 1 *Let* $\overline{c} > \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$ *where* p^* *is given by the solution to equation (9). Then a candidate market equilibrium exists in which firms charge* p^**, a fraction*

$$G\left(\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1] f(\varepsilon) d\varepsilon\right)$$

of consumers checks the products of the two firms and a fraction

$$G\left(\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon\right) - G\left(\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1] f(\varepsilon) d\varepsilon\right)$$

of consumers checks only one product. The rest of the consumers leave the market. For any G*, if* F *is the uniform distribution, an equilibrium surely exists; moreover, if* G *is uniform the equilibrium is unique.*

Proof. See the Appendix.

When $\overline{c}$ is relatively large, the payoff of a firm consists of the sum of the payoff originating from the consumers who check only its product and the payoff stemming from the consumers who check the two products. Under the log-concavity of f, each of these payoffs is quasi-concave (which follows from an application of the Prékopa (1973) aggregation result in our setting). Despite this, unfortunately the sum of these payoffs may fail to be quasi-concave, which implies that we need to impose additional restrictions on the primitives of the model in order to guarantee the existence of a pure-strategy equilibrium.[9] In the Appendix we show that when match values are uniformly

[9] This problem is quite common in search models where demand stems from various consumer types. For example, in the sequential search model of Anderson and Renault (1999) demand stems from consumers who happen to visit a firm for the first time, and from consumers who happen to walk away from a firm and later return to it to conduct a purchase. In their model, assuming that the density of match values f is increasing ensures existence and uniqueness of equilibrium.

distributed, the payoff of a firm is strictly concave in a firm's own price, which guarantees the existence of equilibrium.[10] Uniqueness is guaranteed if search costs are also uniformly distributed.

For arbitrary distributions of match values F and search costs G, the equilibrium may fail to exist. The problem is that an individual firm may find it profitable to deviate from a putative equilibrium price p^* by significantly raising its price, thereby sacrificing profit from the consumers who check the products of the two firms in exchange for profit from the consumers who only check the deviant's product. It is nevertheless possible to provide conditions to rule out such a deviation. Intuitively, it is necessary that the share of consumers who only check one product is not very large. In the working paper version of this article we show that when F is quadratic and convex and G is quadratic and concave, a pure-strategy symmetric equilibrium exists.

3.1 Example: the uniform-uniform case

Consider the following example in which match values and search costs are both uniformly distributed on the unit interval, that is, $\varepsilon \sim U[0,1]$ and $c \sim U[0,1]$. Following Proposition 1, an SNE exists and is unique. In this case, using equations (3) and (4), it is straightforward to derive the cutoffs $c_0(p^*)$ and $c_1(p^*)$:

$$c_0(p^*) = \frac{1}{2}(1-p^*)^2$$

$$c_1(p^*) = \frac{1}{6}(1-p^*)^2(1+2p^*).$$

[10]The strict concavity of the payoff when F is uniform does not carry over to the N-firm model. In Section 5.1 we provide an existence result for the more general oligopoly case.

Using these cutoffs in equation (5) gives the fractions of consumers checking one and two products as a function of the equilibrium price:

$$\mu_1(p^*) = \frac{1}{3}(1-p^*)^3$$
$$\mu_2(p^*) = \frac{1}{6}(1-p^*)^2(1+2p^*).$$

Factoring this optimal consumer search behaviour into the payoff of a deviant firm, the first order necessary condition for a symmetric equilibrium is given by equation (9), which simplifies to:

$$3 - \left(2p^{*2} + p^* + 6\right) p^* = 0.$$

The only real solution to this equation is $p^* \simeq 0.4395$. Plugging this equilibrium price back into the expressions for the fractions of consumers checking one and two products, we conclude that this market has a 0.0586 share of the consumers checking one product and a 0.0983 share checking two products, while the vast majority of consumers does not search any of the products.

4 Higher search costs

We now study how the equilibrium price derived in Proposition 1 depends on the magnitude of search costs. To do this, we parametrise the search cost density by a scalar β and assume that an increase in β shifts the search cost distribution downwards, that is, an increase in β signifies an increase in search costs in the sense of first order stochastic dominance (FOSD). Let $G(c;\beta)$ be a parametrised search cost CDF with $\partial G(c;\beta)/\partial\beta < 0$ and denote the price equilibrium corresponding to a given β by $p^*(\beta)$. We next study how the equilibrium price $p^*(\beta)$ responds to a change in β.

Because we have parametrised G by β, let us denote by $H(p^*;\beta)$ the corresponding parametrised function defined by the FOC in equation (9). By the implicit function theorem, the comparative statics effect of an increase in search costs is then given by

$$\frac{dp^*(\beta)}{d\beta} = -\frac{\partial H}{\partial \beta} \Big/ \frac{\partial H}{\partial p^*}. \tag{11}$$

We have already noted above that the denominator of equation (11), $\partial H/\partial p^*$, is negative. For the numerator of equation (11) we have

$$\frac{\partial H}{\partial \beta} = N(p^*)\frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - D(p^*)\frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta}.$$

Using the equilibrium condition in equation (9), after some manipulations, this expression can be rewritten as:

$$\frac{\partial H}{\partial \beta} = D(p^*)G\left(c_0(p^*), \beta\right)\left[\frac{1}{G\left(c_1(p^*), \beta\right)}\frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - \frac{1}{G\left(c_0(p^*), \beta\right)}\frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta}\right]. \tag{12}$$

The sign of $\partial H/\partial \beta$ is ambiguous; it depends on the values that the hazard rate G'_β/G takes at the cutoff points $c_0(p^*)$ and $c_1(p^*)$, where G'_β is short-hand notation for $\partial G/\partial \beta$. The interesting issue is that this derivative can be negative, in which case the equilibrium price will *decrease* when search costs increase. The next proposition summarises our finding and provides sufficient conditions for the equilibrium price to increase or decrease in search costs. We explain the intuition behind this result after stating it precisely.

PROPOSITION 2 *Let $G(c;\beta)$ be a search cost CDF with positive density on $[0, \overline{c}]$ and with derivative $\partial G(\cdot)/\partial \beta < 0$ so that a higher β signifies higher search costs in the sense of FOSD. Then, the SNE price given by Proposition 1 decreases (increases) in β if and only if*

$$\frac{1}{G\left(c_1(p^*), \beta\right)}\frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - \frac{1}{G\left(c_0(p^*), \beta\right)}\frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta} > (<)0. \tag{13}$$

Moreover, if G'_β/G increases (decreases) in c, then the equilibrium price increases (decreases) in β. The price is independent of β if G'_β/G is constant in c.

Condition (13) is necessary and sufficient for an increase in search costs to result in an increase or decrease in the equilibrium price. Further inspection of this condition reveals that changes in search costs affect both the intensive margin of search (via $G(c_1(p^*))$) and the extensive margin of search (via $G(c_0(p^*))$). At the intensive search margin, the share of consumers who inspect the two products goes down. With fewer consumers comparing the two products, the elasticity of the demand an individual firm faces tends to decrease. However, at the extensive search margin the mass of buyers that check only one of the products also decreases, because the share of consumers that choose to leave the market increases. With fewer consumers checking only one product, the elasticity of the demand an individual firm faces tends to increase. Because the demand of a firm is a weighted average of the demand from these two masses of consumers (those checking one and those checking two products), the elasticity of the demand of an individual firm will increase (and so the equilibrium price will decrease) if and only if the effect of higher search costs on the extensive search margin is stronger than the effect on the intensive search margin. This occurs if and only if the hazard rate G'_β/G evaluated at $c_1(p^*)$ is larger than when evaluated at $c_0(p^*)$.

The result in Proposition 2 is important in that it demonstrates that the standard (positive) association between search costs and prices is based on a restriction on the magnitude of search costs. In fact, in the working paper version of this article, we show that when search costs are restricted to be low, an increase in search costs always results in a higher equilibrium price. The reason for this is that when all consumers check at least one product in equilibrium, a FOSD increase in search costs only affects the intensive search margin. Because overall consumer participation does not change, consumers who stop checking two products end up enlarging the group of buyers who check only one product and, as a result, the demand an individual firm faces becomes less elastic. Correspondingly, firms adjust their prices upwards.

Moraga-González *et al.* (2017a) present a related finding for the standard model of sequential search for differentiated products (cf. Wolinsky, 1986). They also show that, when search costs are not restricted to be low, higher search costs may result in a higher or lower equilibrium price and provide necessary and sufficient conditions for these effects to occur. Mathematically, the conditions they give are however quite different from those in condition (13).[11] The main distinction relates to the nature of search in the two different models. While with simultaneous search what matters for pricing are the relative masses of consumers checking one or two products, with sequential search the entire density of the various consumer types who search in the market affects price. As a result, with simultaneous search the conditions involve the behaviour of the search cost CDF with respect to the search cost shifter while with sequential search the conditions refer to the behaviour of the search cost PDF.[12]

[11]As a matter of fact, in the working paper version of this article we provide examples showing that an FOSD increase in search costs may cause the equilibrium price to decrease with simultaneous search and to increase with sequential search, and the other way around. This observation is of crucial importance for the empirical researcher because counterfactual analysis of lower search costs may lead to wrong conclusions if the consumer search protocol is mis-specified.

[12]See also the online Appendix of Moraga-González *et al.* (2017a) for a study of simultaneous search for homogeneous products (cf. Burdett and Judd, 1983), as well as Fabra and Reguant (2018), who study price discrimination in such a setting. It is reassuring to learn that, when not all consumers search, the insight that prices can increase or decrease in search costs is robust to the search protocol (sequential vs. simultaneous), the type of product (differentiated vs. homogeneous) and the nature of the market equilibrium (pure vs. mixed strategies).

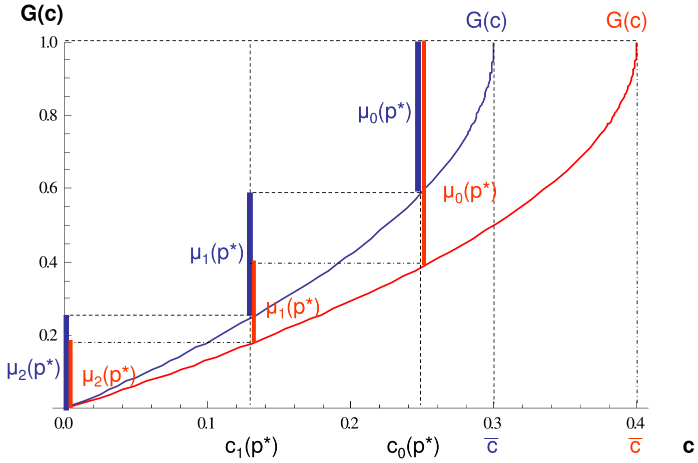

Figure 2: *An Increase In Search Costs Leading To A Lower Equilibrium Price*

The result that the equilibrium price may decrease in search costs is illustrated in Figure 2. In this figure we represent the effect of an increase in search costs on the intensive and extensive search margins. Initially, consumer search costs are given by the left search cost distribution.[13] The increase in search costs is represented by the shift from the left distribution to the right one. As the graph shows, the increase in search costs is much more felt at the higher percentiles of the search cost distribution. As a result, when search costs increase, keeping the equilibrium price fixed, the share of consumers who do not search at all increases substantially. This causes the share of inelastic consumers to fall significantly more than the share of elastic consumers; this demand composition effect increases the overall elasticity of demand, and the equilibrium price falls as a result.

Proposition 2 also provides a sufficient condition for the equilibrium price to increase or decrease when search costs go up. The equilibrium price surely increases in search costs when the hazard

[13]We use the Kumaraswamy (1980) distribution for G in Figure 2, with parameters $a = 1$, $b = 1/2$, and upper bound $\beta = 0.3$. In Section 4.2 we formally introduce this distribution and demonstrate that G'_β/G is increasing in c for those parameter values.

rate G'_β/G (or the elasticity of G with respect to β) is increasing in search costs. In such a case, an increase in search costs is more noticeable at lower percentiles of the search cost distribution than at higher, which implies that the effect on the intensive search margin is stronger than the effect on the extensive search margin. Alternatively, when the hazard rate G'_β/G is decreasing, higher search costs result in a lower equilibrium price. At the end of Section 4.1 we relate these sufficient conditions to the notions of log-supermodularity and log-submodularity of distribution functions.

4.1 The reversed hazard rate stochastic ordering

In this section we relate the sufficient condition in Proposition 2 to the *reversed hazard rate ordering* of distributions (see Shaked and Shanthikumar, 2007).

DEFINITION 1 *The distribution $G(c;\beta)$ has the increasing reversed hazard rate (IRHR) property if and only if for any $\beta' < \beta$,*

$$G(c,\beta)G(d,\beta') \leq G(c,\beta')G(d;\beta)$$

for any $c \leq d$ in the union of the supports of $G(c,\beta')$ and $G(c;\beta)$.

If the reverse property holds, $G(c;\beta)$ has *decreasing reversed hazard rates*:

DEFINITION 2 *The distribution $G(c;\beta)$ has the decreasing reversed hazard rate (DRHR) property if and only if for any $\beta' < \beta$,*

$$G(c,\beta)G(d,\beta') \geq G(c,\beta')G(d;\beta)$$

for any $c \leq d$ in $[0, \min\{\overline{c}(\beta), \overline{c}(\beta')\}]$.[14]

[14]Note that we define DRHR up to the minimum of the upper bounds of the supports of the search cost distributions $G(c,\beta)$ and $G(c,\beta')$. This is needed for compatibility of the DRHR ranking with the FOSD ranking of distributions.

A few simple calculations reveal that for distributions with IRHR, the ratio G'_β/G increases in c, which is equivalent to the notion of *log-supermodularity* of the distribution function. On the contrary, for distributions with DRHR, the ratio G'_β/G decreases in c; this is then equivalent to *log-submodularity* of the distribution function.

Corollary 1 to Proposition 2. *For log-supermodular (log-submodular) search cost distributions, an FOSD increase in search costs results in an increase (decrease) in the equilibrium price.*

We note that the notions of IRHR (log-supermodularity) and DRHR (log-submodularity) take very simple forms in the common cases of additive and multiplicative shocks to search costs. In the case of multiplicative shocks, the search cost distribution is $G(c/(1+\beta))$, with $\beta \geq 0$. In this case, IRHR (DRHR) is identical to cg/G being decreasing (increasing), which is the same as decreasing (increasing) search cost elasticity of the cumulative distribution function G. In the case of additive shocks, the search cost distribution is $G(c-\beta)$, with $\beta \geq 0$. Redefining the notion of DRHR on the set $[\max\{\underline{c}(\beta), \underline{c}(\beta')\}, \min\{\overline{c}(\beta), \overline{c}(\beta')\}]$ we note then that IRHR (DRHR) is equivalent to g/G being decreasing (increasing), which is the same as log-concavity (log-convexity) of the distribution function G.[15]

[15]In Moraga-González *et al.*'s (2017a) model of sequential search for differentiated products, conditions based on the likelihood ratio ranking of densities are provided for a similar implication. Note that the likelihood ratio ordering, which relates to the density functions of random variables, is stronger than the reversed hazard rate ordering, which has to do with the cumulative distribution functions. A consequence of this distinction is that while our results here apply to the case of additive search costs, this is not so in the model of sequential search.

4.2 An illustrative example: The Kumaraswamy's distribution

The Kumaraswamy's (1980) distribution has a CDF G and a PDF g given by:

$$\begin{aligned} G(c) &= 1 - \left[1 - \left(\frac{c}{\beta}\right)^a\right]^b, \ c \in [0, \beta], \quad a, b > 0; \\ g(c) &= \frac{ab}{\beta}\left(\frac{c}{\beta}\right)^{a-1}\left[1 - \left(\frac{c}{\beta}\right)^a\right]^{b-1}. \end{aligned}$$

The Kumaraswamy distribution is often used as a substitute for the beta distribution (see Ding and Wolfstetter, 2011). An increase in β signifies an increase in search costs for all consumers. Depending on the parameter b, this distribution can be log-supermodular ($b > 1$) or log-submodular ($0 < b < 1$). For this distribution we get the following result:

Corollary 2 to Proposition 2 *Assume that search costs are distributed on the interval* $[0, \beta]$ *according to the Kumaraswamy distribution. Then, for all* a*, the equilibrium price decreases in* β *if* $0 < b < 1$*, is constant in* β *if* $b = 1$*, and increases in* β *if* $b > 1$.

5 Extensions

In this section, we look at two extensions of our main model: the case of more than two firms and a model in which firms differ in the probability of being sampled by consumers.

5.1 The N-firm model

The previous simultaneous search model with differentiated products can easily be generalised to the case of $N > 2$ firms. In addition, our results about the effect of higher search costs on the equilibrium price hold in more general oligopolies.

The problem of a consumer with search cost c is to choose a number k of firms to be sampled in order to maximise in k her expected utility:

$$U(k,c) = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) k F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon - kc.$$

It can easily be checked that this problem is well behaved and that a unique solution exists. Such a solution defines a partition of the consumer population into groups of buyers $\mu_k(p^*)$ that search $k = 0, 1, 2, \ldots, N$ firms, with $\sum_{k=0}^{N} \mu_k(p^*) = 1$.

The sizes of these groups are given by the expressions:[16]

$$\begin{aligned} \mu_0 &= 1 - G(c_0(p^*)) \\ \mu_k &= G(c_{k-1}(p^*)) - G(c_k(p^*)),\ k = 1, 2, \ldots, N-1 \\ \mu_N &= G(c_{N-1}(p^*)) - G(c_N(p^*)) = G(c_{N-1}(p^*)) \text{ since } c_N = 0. \end{aligned} \tag{14}$$

where

$$\begin{aligned} c_0(p^*) &= \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \\ c_k(p^*) &= \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(k+1)F(\varepsilon) - k\right] F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon,\ k = 1, 2, \ldots, N-1. \end{aligned}$$

The expected payoff of a firm i that deviates from the symmetric equilibrium price by charging a price $p_i \neq p^*$ is

$$\pi_i(p_i; p^*) = (p_i - r) \left(\frac{\mu_1(p^*)}{2} \Pr[\varepsilon_i \geq p_i] + \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \Pr\left[\varepsilon_i - p_i \geq \max\{z_{k-1} - p^*, 0\}\right] \right), \tag{15}$$

where $z_k \equiv \max\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k\}$. As before, the demand of the deviant firm i stems from the various consumer groups, and a consumer who searches k times compares the offer of firm i with the offers of $k-1$ other firms.

[16] As in the analysis of the duopoly case above, when the upper bound of the search cost distribution is not sufficiently large, some of these groups may have zero mass. In line with the analysis above, we ignore those cases here and refer the reader to the working paper version of this article for further details.

Taking the FOC, imposing symmetry, simplifying and rearranging we obtain:

$$\begin{aligned}\mu_1(p^*)\left[1-F(p^*)-(p^*-r)f(p^*)\right]&+\sum_{k=2}^{N}k\mu_k(p^*)\int_{p^*}^{\overline{\varepsilon}}F(\varepsilon)^{k-1}f(\varepsilon)d\varepsilon\\ -(p^*-r)\sum_{k=2}^{N}k\mu_k(p^*)&\left(\int_{p^*}^{\overline{\varepsilon}}(k-1)F(\varepsilon)^{k-2}f(\varepsilon)^2d\varepsilon+F(p^*)^{k-1}f(p^*)\right)=0. \qquad (16)\end{aligned}$$

In the Appendix we show that a candidate equilibrium $p^* \in [r, p^m]$ that solves this equation exists.

For the candidate price p^* to be a SNE, the payoff function in equation (15) must be quasi-concave in p_i. Using the well-known aggregation result of Prékopa (1973), it can be shown that each of the summands of the payoff function in equation (15) is quasi-concave. However, even if each element of the sum of payoffs is quasi-concave, the payoff function need not be quasi-concave. Building on a recent contribution by Choi and Smith (2017), we can prove the following result.

PROPOSITION 3 *Let $\overline{c} > \int_{p^*}^{\overline{\varepsilon}}(\varepsilon - p^*)f(\varepsilon)d\varepsilon$ where p^* is given by the solution to equation (16). Then a candidate market equilibrium exists in which firms charge p^* and a fraction $\mu_k(p^*)$, $k = 0, 1, 2, ..., N$ of consumers checks the products of k firms, where the fractions $\mu_k(p^*)$ are given by (14).*

Suppose that the search cost distribution G is arbitrary and the distribution of match values F is uniform. Then if $N < 9$ an equilibrium surely exists, while if N is arbitrary an equilibrium surely exists whenever $r > (N-3)/((N+1)\overline{\varepsilon})$.

For the proof of this result, we refer the reader to the working paper version of this article. Our proof builds on the novel insight by Choi and Smith (2017) that the weighted sum of quasi-concave functions is also quasi-concave if the increasing part of each is more risk averse than any decreasing part. To apply this result in our setting, we first verify that each of the summands of the payoff function in equation (15) is quasi-concave. After this, for two arbitrary summands, we identify the set of prices for which one summand is increasing and the other is decreasing. Finally, we show that Choi and Smith's condition holds when either the number of firms is sufficiently low or the marginal cost is sufficiently large.[17]

5.2 Non-uniform sampling

Hortaçsu and Syverson (2004), De los Santos *et al.* (2012), and De los Santos (2018) provide empirical evidence that some firms are more salient than others and because of this consumers are more likely to encounter them when searching for products. In this section, we explore the implications of non-uniform sampling for pricing, firm profits, and consumer surplus. In addition, we show that our results about the effect of higher search costs on the equilibrium price do not qualitatively depend on the assumption that firms are equally likely to be sampled.

Assume that one of the firms, say firm 1, is more likely to be sampled than the other firm. Let α be the probability with which a consumer who searches once comes across the offer of firm 1, where $\alpha \geq 1/2$. Correspondingly, $1-\alpha$ is the probability with which a consumer who searches once finds the offer of firm 2.. Notice that the case of $\alpha = 1/2$ corresponds to the symmetric model we have analysed in Section 3. Following Armstrong *et al.* (2009), in what follows we focus on the case of $\alpha = 1$. The more general case in which $\alpha \geq 1/2$ can be consulted in the working paper version of this article.

Let p_1^* and p_2^* be the equilibrium prices of the firms. Because $\alpha = 1$, the firms will have asymmetric

[17] In the working paper version of this article, we provide evidence based on numerical solutions of the N-firm model using the Kumaraswamy distribution that higher search costs may result in a lower, equal, or higher equilibrium price, which is the same result as in the duopoly case discussed in Section 4.2.

demands so we expect these prices to be different from one another.

We next characterise optimal consumer search behaviour. Proceeding as in the previous section, a consumer with search cost c that chooses to inspect only one product derives an expected utility equal to:

$$U(1,c) = \Pr\left[\varepsilon_1 \geq p_1^*\right] E\left[\varepsilon_1 - p_1^* | \varepsilon_1 \geq p_1^*\right] - c, \tag{17}$$

If, instead, the consumer with search cost c chooses to inspect the products of the two firms, his/her expected utility is:

$$U(2,c) = Pr[\max\{\varepsilon_1 - p_1^*, \varepsilon_2 - p_2^*\} \geq 0] E[\max\{\varepsilon_1 - p_1^*, \varepsilon_2 - p_2^*\} | \max\{\varepsilon_1 - p_1^*, \varepsilon_2 - p_2^*\} \geq 0] - 2c. \tag{18}$$

Note that equation (17) is virtually the same as equation (1) although we rewrite it here to emphasise that with prominence consumers who choose to search one time will check the product of firm 1. The main difference between equations (18) and (2) is that the two equilibrium prices p_1^* and p_2^* now play a role.

Equating equation (17) to zero gives the cutoff c_0 above which consumers will not check any product:

$$c_0(p_1^*) \equiv \int_{p_1^*}^{\bar{\varepsilon}} (z - p_1^*) f(z) dz.$$

Equating equation (17) to equation (18) gives the critical search cost value above which it is worth to search once and not twice:

$$c_1(p_1^*, p_2^*) \equiv \int_0^{\bar{\varepsilon} - p_2^*} z[f(z + p_1^*)F(z + p_2^*) + F(z + p_1^*)f(z + p_2^*)]dz - \int_0^{\bar{\varepsilon} - p_1^*} z f(z + p_1^*) dz,$$

where we have assumed that $p_1^* > p_2^*$, something that we later check it holds in equilibrium.

The shares of consumers searching for one and two products are then:

$$\mu_1(p_1^*, p_2^*) = G(c_0(p_1^*)) - G(c_1(p_1^*, p_2^*)) \text{ and } \mu_2(p_1^*, p_2^*) = G(c_1(p_1^*, p_2^*)). \tag{19}$$

We now move to the problem of the firms. Consider first firm 1, the prominent firm. The expected payoff to firm 1 when deviating by charging a price $p_1 \neq p_1^*$ is:

$$\pi_1(p_1; p_1^*, p_2^*) = (p_1 - r)\left(\frac{\mu_1(p_1^*, p_2^*)}{2}\Pr[\varepsilon_1 \geq p_1] + \mu_2(p_1^*, p_2^*)\Pr\left[\varepsilon_1 - p_1 \geq \max\{\varepsilon_2 - p_2^*, 0\}\right]\right). \tag{20}$$

Consider now firm 2, the non-prominent firm. The expected payoff to this firm when deviating by charging a price $p_2 \neq p_2^*$ is:

$$\pi_2(p_2; p_1^*, p_2^*) = (p_2 - r)\mu_2(p_1^*, p_2^*)\Pr\left[\varepsilon_2 - p_2 \geq \max\{\varepsilon_1 - p_1^*, 0\}\right]. \tag{21}$$

Computing a price equilibrium requires solving the system of FOCs corresponding to the payoffs in equations (20)–(21) for p_1^* and p_2^*, after factoring the expressions for $\mu_1(p_1^*, p_2^*)$ and $\mu_2(p_1^*, p_2^*)$ given in equation (19). Unfortunately, the resulting system of equations is extremely complicated to deal with. To make further progress, we assume that match values and search costs are uniformly distributed. In particular, let us assume that $\varepsilon \sim U[0, 1]$ and $c \sim U[0, \overline{c}]$.

PROPOSITION 4 *Assume that one of the firms, say firm 1, is prominent. Also assume that match values and search costs are uniformly distributed on* $[0, 1]$ *and* $[0, \overline{c}]$*, respectively. Then, for any*

$$\overline{c} \in \left(\frac{8\sqrt{2} - 11}{3}, \frac{1}{8} + \frac{1}{18\sqrt{3}}\right],$$

there exists a unique price equilibrium (p_1^*, p_2^*) *in pure-strategies. The equilibrium prices satisfy the inequality*

$$p_2^* < p_1^* < p^*,$$

where p^* is the symmetric equilibrium price. As a result, market prominence increases consumer surplus.

For the proof of this result, we refer the reader to the working paper version of this article. Proposition 4 shows that prominence increases consumer surplus. The intuition behind this result is as follows. When all the consumers who inspect just one product visit the prominent firm, this firm's pool of consumers becomes less elastic compared to the symmetric equilibrium situation. As a result, everything else equal, a firm that becomes salient in the market tends to increase its price. By contrast, the non-prominent firm's pool of consumers becomes more elastic compared to the symmetric equilibrium because this firm only attracts consumers who are willing to inspect the products of the two firms. Thus, for this firm the situation is the opposite and hence it tends to decrease its price. Proposition 4 shows that the decrease in the non-prominent firm's price is so strong that the prominent firm also decreases its price relative to the symmetric equilibrium price of Section 3. As a result, consumer surplus increases.

The impact of prominence in our model with simultaneous search is quite different from its impact when consumers search sequentially. Armstrong *et al.* (2009) show that the prominent firm charges a lower price than the non-prominent firm; moreover, they show that prominence typically results in a decrease in consumer surplus. The difference in results is intimately linked to the nature of consumer search. When consumers search sequentially, they first check the product of the prominent firm. The mere fact that a consumer arrives at the non-prominent firm then reveals that the match with the product of the prominent firm is poor. This gives the non-prominent firm an incentive to raise its price over and above the price of the prominent firm. Compared to the symmetric case of uniform sampling, the prominent firm has disproportionally more fresh demand and the non-prominent firm disproportionally more returning demand. As a result, the price of the prominent firm is below the symmetric equilibrium price and the price of the non-prominent firm above. This results in a lower consumer surplus compared to uniform sampling.

The insight in Proposition 4 that consumers gain from non-uniform sampling situations is not unique to the case $\alpha = 1$. In the working paper version of this article we show numerically that the same result arises for the entire range $\alpha \in (1/2, 1]$. The only difference worth mentioning is that when α is relatively low the equilibrium price of the prominent firm is higher than the symmetric equilibrium price with uniform sampling. Despite this, consumer surplus is higher. We also show numerically that the effects of higher search costs are the same as with uniform sampling.

6 Conclusions

This paper has extended the literature on simultaneous search by allowing for differentiated products and consumer search cost heterogeneity. While such a framework has recently received empirical support and has thus been the basis for a number of current empirical applications, with the exception of Anderson, De Palma, and Thisse (1992), models of simultaneous search for differentiated products have not received much attention in the theoretical literature to date.

In contrast to Anderson, De Palma, and Thisse (1992) where all consumers have the same search cost, in our paper consumers choose to inspect different numbers of products before buying. The consumer equilibrium is thus a partition of the set of consumers into subsets of buyers inspecting k products, $k = 0, 1, 2, \ldots, N$. Consequently, the aggregate demand of a typical firm stems from the demands of these distinct consumer groups. The more products inspected, the more price sensitive the consumer is. Absent the possibility of price discrimination, this poses a complicated pricing problem. We have shown that when the distribution of search costs is arbitrary, the pricing problem is well behaved and a pure strategy symmetric Nash equilibrium exists in this model when the distribution of match values is uniform and the number of competitors is lower than nine. For an arbitrary number of rival firms, a marginal cost of production large enough suffices for the existence of a symmetric equilibrium in pure strategies.

We have also studied the effects of increasing search costs on the equilibrium price. The typical assumption in the existing literature is that all consumers search or, equivalently, that all consumers have sufficiently low search costs. With arbitrary search cost heterogeneity, this assumption is, at the very least, questionable. We have shown that, depending on the nature of the search cost distribution, an increase in the search costs of all consumers may result in a lower or in a higher equilibrium price. The key to understanding this result is to recognise that an increase in search costs affects two margins that influence the elasticity of demand. When search costs increase, consumers check fewer products before buying. This tends to decrease the elasticity of demand. However, when search cost increase, some consumers who used to search cease to search altogether. This tends to increase the elasticity of demand because the consumers who quit the market are the high-search-cost ones. We have first derived a necessary and sufficient condition for the equilibrium price to decrease (increase) in search costs. We have then shown that for distributions with the decreasing (increasing) reversed hazard rate property, which is equivalent to log-submodularity (log-supermodularity) of the cumulative distribution function, the equilibrium price will decrease (increase) as the costs of search of all consumers rise.

We have finally examined the effects of non-uniform sampling, that is, the idea that some firms are more salient than others in the marketplace and therefore their products are more likely to be inspected by consumers than those of the rest of the firms. We have focused on the special case in which one of the firm products is prominent, that is, it is always inspected by the consumers who choose to inspect only one item. We have shown that the prominent firm charges a higher price than the rival firm. Moreover, the equilibrium prices of both firms are below the symmetric equilibrium price. Thus, market prominence works in favor of consumers. These results generalise to weaker forms of market saliency.

Appendix

Proof of Proposition 1. First we show that a candidate equilibrium price $p^* \in [r, p^m]$ exists. We observe first that when we evaluate the function H at $p = r$ we obtain

$$H\left(r\right) = \left(1 - F(r)\right)\left[F(r)G\left(c_1(r)\right) + G\left(c_0(r)\right)\right] > 0.$$

Second, if we evaluate it at $p = p^m$ then we get that

$$H\left(p^m\right) = N(p^m)G\left(c_1(p^m)\right), \tag{22}$$

just because the price p^m satisfies the first order condition for the monopoly problem: $1 - F(p^m) - (p^m - r)\, f(p^m) = 0$. The sign of $H(p^m)$ depends on the sign of $N(p^m)$, which, after simplification, can be written as:

$$N(p^m) = (p^m - r)\left[f(p^m)\left[1 - F(p^m)\right] - 2\int_{p^m}^{\overline{\varepsilon}} f\left(\varepsilon\right)^2 d\varepsilon\right]. \tag{23}$$

The sign of this expression depends on the sign of the term inside the squared brackets. Let us define it as:

$$M(p) \equiv f(p)\left[1 - F(p)\right] - 2\int_p^{\overline{\varepsilon}} f(\varepsilon)^2\, d\varepsilon.$$

Taking the derivative of M with respect to p gives $f'(p)(1 - F(p)) + f(p)^2$, which is greater than zero by log-concavity of f (see Corollary 2 in Bagnoli and Bergstrom, 2005). Since M is increasing in p and is equal to zero when we set $p = \overline{\varepsilon}$, we conclude that $M(p^m) < 0$. Hence $H(p^m) < 0$. Since H is a continuous function with $H(r) > 0$ and $H(p^m) < 0$, we conclude that for any log-concave density f, there exists a candidate price equilibrium $p^* \in [r, p^m]$. Note also that at the candidate equilibrium price p^* we must have $dH(p^*)/dp < 0$.

We now show that when the match values follow a uniform distribution, the payoff function in equation (8) is strictly concave. For the uniform distribution, we have $F(\varepsilon) = \varepsilon/\overline{\varepsilon}$, $f(\varepsilon) = 1/\overline{\varepsilon}$, and $f'(\varepsilon) = 0$. The second order derivative of equation (8) is:

$$\begin{aligned}
\frac{d^2\pi_i\left(p_i > p^*\right)}{dp_i^2} &= -\frac{\mu_1(p^*)}{\overline{\varepsilon}} - 2\mu_2(p^*)\left(\frac{\overline{\varepsilon} - p_i}{\overline{\varepsilon}^2} + \frac{p^*}{\overline{\varepsilon}^2}\right) + \left(p_i - r\right)\mu_2(p^*)\frac{1}{\overline{\varepsilon}^2}; \\
&= -\frac{\overline{\varepsilon}\mu_1(p^*) + \mu_2(p^*)(2\overline{\varepsilon} - 3p_i + 2p^* + r)}{\overline{\varepsilon}^2},
\end{aligned}$$

which is clearly negative because p_i cannot be greater than the monopoly price, which in this case of the uniform distribution is given by $p^m = (\overline{\varepsilon} + r)/2$. In a similar way, we can compute the second order condition for prices $p_i < p^*$, which gives

$$\frac{d^2\pi_i\left(p_i < p^*\right)}{dp_i^2} = -\frac{1}{\overline{\varepsilon}}\left(\mu_1(p^*) + 2\mu_2(p^*)\right) < 0.$$

Because of strict concavity of the payoff function in own price p_i, the candidate equilibrium price is indeed a symmetric Nash equilibrium.

Finally, we remark that for consumers to search as prescribed in the result, we need that $c_0(p^*) < \overline{c}$, which gives the condition in the Proposition.

For the uniqueness result, notice that the function H can be written as

$$H(p) = \frac{\overline{\varepsilon}r - p^2}{\overline{\varepsilon}^2} G\left(\frac{(\overline{\varepsilon} - p)^2(\overline{\varepsilon} + 2p)}{6\overline{\varepsilon}^2}\right) + \frac{\overline{\varepsilon} - 2p + r}{\overline{\varepsilon}} G\left(\frac{(\overline{\varepsilon} - p)^2}{2\overline{\varepsilon}}\right).$$

for the case where F is the uniform distribution. When search costs are also uniform, this simplifies to

$$H(p) = \frac{(\overline{\varepsilon} - p)^2}{2\overline{\varepsilon}^2\overline{c}} \left(\frac{(\overline{\varepsilon}r - p^2)(\overline{\varepsilon} + 2p)}{3\overline{\varepsilon}^2} + \overline{\varepsilon} - 2p + r\right).$$

The candidate equilibrium solves $(\overline{\varepsilon}r - p^2)(\overline{\varepsilon} + 2p)/(3\overline{\varepsilon}^2) + \overline{\varepsilon} - 2p + r = 0$. This expression is monotone decreasing because its derivative is $-2 - 2(p - r)/(3\overline{\varepsilon}) - 6p^2/(3\overline{\varepsilon}^2)$, which is always negative. As a result, $H(p)$ only crosses the horizontal axes once.

Proof of Corollary of Proposition 2. We show that, for any a and β, the Kumaraswamy distribution is log-supermodular for $b > 1$ and log-submodular for $0 < b < 1$. The case of $b = 1$ is special in that the distribution is both log-supermodular and log-submodular; in that case the equilibrium price remains constant as search costs increase.

Note that for the Kumaraswamy distribution it holds that

$$\frac{\partial G(c;\beta)}{\partial \beta} = -\frac{ab}{\beta}\left(\frac{c}{\beta}\right)^a \left(1 - \left(\frac{c}{\beta}\right)^a\right)^{b-1} < 0;$$

correspondingly, the hazard ratio G'_β/G is

$$\frac{G'_\beta(c;\beta)}{G(c;\beta)} = -\frac{\frac{ab}{\beta}\left(\frac{c}{\beta}\right)^a \left(1 - \left(\frac{c}{\beta}\right)^a\right)^{b-1}}{1 - \left[1 - \left(\frac{c}{\beta}\right)^a\right]^b}. \tag{24}$$

We now let

$$t \equiv 1 - \left(\frac{c}{\beta}\right)^a.$$

Note that $t \in (0,1)$ and that t is monotonically decreasing in c. We can rewrite equation (24) as

$$\frac{G'_\beta}{G} = -\frac{ab(1-t)t^{b-1}}{\beta(1-t^b)},$$

and then take the derivative of G'_β/G with respect to t. This gives

$$\frac{d[G'_\beta/G]}{dt} = -\frac{abt^{b-2}(b-1-bt+t^b)}{\beta(1-t^b)^2}.$$

We now argue that this derivative is negative for all $b > 1$ and positive for all $0 < b < 1$. Consider first the $b > 1$ case. Let $h(t) \equiv b - 1 - bt + t^b$. Then $h(0) = b - 1 > 0$, $h(1) = 0$, and $h'(t) = -b\left(1 - t^{b-1}\right) < 0$. So h is monotonically decreasing and hence $h(t) > 0$ for any $t \in (0,1)$. As a result, G'_β/G decreases in t (and thus increases in c). By Proposition 2, this implies that for the Kumaraswamy family of search cost distributions with parameter $b > 1$, the equilibrium price increases as search costs rise.

Second, assume $0 < b < 1$. In this case we have $h(0) = b - 1 < 0$, $h(1) = 0$ and $h'(t) = -b\left(1 - t^{b-1}\right) > 0$. Hence $h(t) < 0$ for any $t \in (0,1)$. As a result, G'_β/G increases in t (and therefore decreases in c). By Proposition 2, this implies that for the Kumaraswamy family of search cost distributions with parameter $0 < b < 1$, the equilibrium price decreases as search costs go up.

For completeness, let $b = 1$. Plugging $b = 1$ in equation (24) gives $G'_\beta/G = -a/\beta$, which is constant in c and therefore the equilibrium price does not vary with β.

Vrije Universiteit Amsterdam and University of Groningen

Sapientia Hungarian University of Transylvania

Kelley School of Business, Indiana University

References

[1] Anderson, S.P., de Palma, A. and Thisse, J.F. (1992). *Discrete Choice Theory of Product Differentiation*, The MIT Press, Cambridge.

[2] Anderson, S.P. and Renault, R. (1999). 'Pricing, product diversity, and search costs: a Bertrand-Chamberlin-Diamond model', *RAND Journal of Economics,* vol. 30, pp. 719-35.

[3] Anderson, S.P. and Renault, R. (2018). 'Firm pricing with consumer search', in Corchón, L.C. and Marini, M.A. eds.) *Handbook of Game Theory and Industrial Organization,* vol. II, pp. 177-224, Edward Elgar Publishing Inc.

[4] Armstrong, M., Vickers, J., and Zhou, J. (2009). 'Prominence and consumer search', *RAND Journal of Economics,* vol. 40, pp. 209-233.

[5] Armstrong, M. and Zhou, J. (2011). 'Paying for prominence', *Economic Journal,* vol. 121(556), pp. F368-F395.

[6] Bagnoli, M. and Bergstrom, T. (2005). 'Log-concave probability and its applications', *Economic Theory,* vol. 26, pp. 445-469.

[7] Burdett, K. and Judd, K.L. (1983). 'Equilibrium price dispersion', *Econometrica,* vol. 51, pp. 955-969.

[8] Choi, M. and Smith, L. (2017). 'Ordinal aggregation results via Karlin's variation diminishing property', *Journal of Economic Theory,* vol. 168, pp. 1-11.

[9] De los Santos, B. (2018). 'Consumer search on the Internet', *International Journal of Industrial Organization,* vol. 58, pp. 66-105.

[10] De los Santos, B., Hortaçsu, A. and Wildenbeest, M.R. (2012). 'Testing models of consumer search using data on web browsing and purchasing behaviour', *American Economic Review,* vol. 102, pp. 2955-2980.

[11] Diamond, P.A. (1971). 'A Model of price adjustment', *Journal of Economic Theory* vol. 3, pp. 156-168.

[12] Ding, W. and Wolfstetter, E. (2011). 'Prizes and lemons: procurement of innovation under imperfect commitment', *RAND Journal of Economics,* vol. 42, pp. 664-680.

[13] Donna, J.D., Pereira, P., Pires, T. and Trindade, A. (2018). 'Measuring the welfare of intermediation in vertical markets', Working Paper.

[14] Ershov, D. (2018). 'The Effects of consumer search costs on entry and quality in the mobile app market', Working Paper.

[15] Fabra, N. and Reguant, M. (2018). 'A model of search with price discrimination', Working Paper.

[16] Fishman, A. and Lubensky, D. (2018). 'Search prominence and return costs', *International Journal of Industrial Organization,* vol. 58, pp. 136-161.

[17] Haan, M.A. and Moraga-González, J.L. (2011). 'Advertising for attention in a consumer search model', *Economic Journal,* vol. 121(552), pp. F552-F579.

[18] Hong, H. and Shum, M. (2006). 'Using price distributions to estimate search costs', *RAND Journal of Economics,* vol. 37, pp. 257-275.

[19] Honka, E. (2014). 'Quantifying search and switching costs in the US auto insurance industry', *RAND Journal of Economics,* vol. 45, pp. 847-884.

[20] Honka, E. Chintagunta, P. (2017). 'Simultaneous or sequential? Search strategies in the US auto insurance industry', *Marketing Science,* vol. 36, pp. 21-40.

[21] Hortaçsu, A. and Syverson, C. (2004). 'Product differentiation, search costs, and competition in the mutual fund industry: a case study of S&P 500 index funds." *Quarterly Journal of Economics*, vol. 119, pp. 403-456.

[22] Janssen, M.C.W. and Moraga-González, J.L. (2004). 'Strategic pricing, consumer search and the number of firms', *Review of Economic Studies,* vol. 71, pp. 1089-118.

[23] Kumaraswamy, P. (1980). 'A generalised probability density function for double-bounded random-processes', *Journal of Hydrology,* vol. 46, pp. 79-88.

[24] Lin, H. and Wildenbeest, M.R. (2019). 'Non-parametric estimation of search costs for differentiated products: evidence for Medigap', *Journal of Business & Economic Statistics*, forthcoming.

[25] McDevitt, R. (2014). "'A" business by any other name: firm name choice as a signal of firm quality', *Journal of Political Economy,* vol. 122, pp. 909-944.

[26] Moraga-González and V. Petrikaitė (2013). 'Search costs, demand-side economies and the incentives to merge under Bertrand competition', *RAND Journal of Economics* vol. 44, pp. 391-424.

[27] Moraga-González, J.L. and Wildenbeest, M.R. (2008). 'Maximum likelihood estimation of search costs', *European Economic Review,* vol. 52, pp. 820-48.

[28] Moraga-González, J.L., Sándor, Z. and Wildenbeest, M.R. (2013). 'Semi-nonparametric estimation of consumer search costs', *Journal of Applied Econometrics,* vol. 28, pp. 1205-1223.

[29] Moraga-González, J.L., Sándor, Z. and Wildenbeest, M.R. (2015). 'Consumer search and prices in the automobile market', CEPR Discussion Paper DP10487.

[30] Moraga-González, J.L., Sándor, Z. and Wildenbeest, M.R. (2017a). 'Prices and heterogeneous search costs', *RAND Journal of Economics,* vol. 48, pp. 125-146.

[31] Moraga-González, J.L., Sándor, Z. and Wildenbeest, M.R. (2017b). 'Nonsequential search equilibrium with search cost heterogeneity', *International Journal of Industrial Organization,* vol. 50, pp. 392-414.

[32] Moraga-González, J.L., Sándor, Z. and Wildenbeest, M.R. (2020). 'Simultaneous search for differentiated products: the impact of search costs and firm prominence', CEPR Discussion Paper DP14454.

[33] Morgan, P. and Manning, R. (1985). 'Optimal search', *Econometrica,* vol. 53, pp. 923-944.

[34] Murry, C. and Zhou, Y. (2019). 'Consumer search and automobile dealer co-location', *Management Science*, forthcoming.

[35] Perloff, J.M. and Salop, S.C. (1985). 'Equilibrium with product differentiation', *Review of Economic Studies* vol. 52, pp. 107-120.

[36] Pires, T. (2016). 'Costly search and consideration sets in storable goods markets', *Quantitative Marketing and Economics,* vol. 14, pp. 157-193.

[37] Pires, T. (2018). 'Measuring the effects of search costs on equilibrium prices and profits', *International Journal of Industrial Organization,* vol. 60, pp. 179-205.

[38] Prékopa, A. (1973). 'On logarithmic concave measures and functions', *Acta Scientiarum Mathematicarum,* vol. 34, pp. 335-343.

[39] Rhodes, A. (2011). 'Can prominence matter even in an almost frictionless market', *Economic Journal,* vol. 121(556), pp. 297-308.

[40] Sanches, F., Silva Junior, D. and Srisuma, S. (2018). 'Minimum distance estimation of search costs using price pistribution', *Journal of Business & Economic Statistics*, vol. 36, pp. 658-671.

[41] Stahl, D.O. (1989). 'Oligopolistic pricing with sequential consumer search', *American Economic Review,* vol. 79, pp. 700-12.

[42] Stahl, D.O. (1996). 'Oligopolistic pricing with heterogeneous consumer search', *International Journal of Industrial Organization,* vol. 14, pp. 243-268.

[43] Stigler, G. (1961). 'The economics of information', *Journal of Political Economy*, vol. 69, pp. 213-225.

[44] Weitzman, M.L. (1979). 'Optimal search for the best alternative', *Econometrica,* vol. 47-3, pp. 641-654.

[45] Wildenbeest, M.R. (2011). 'An empirical model of search with vertically differentiated products', *RAND Journal of Economics,* vol. 42, pp. 729-57.

[46] Wilson, C.M. (2010). 'Ordered search and equilibrium obfuscation', *International Journal of Industrial Organization,* vol. 28, pp. 496-506.

[47] Wolinsky, A. (1983). 'Retail trade concentration due to consumers' imperfect information', *The Bell Journal of Economics,* vol. 14, pp. 275-282.

[48] Wolinsky, A. (1986). 'True monopolistic competition as a result of imperfect information', *Quarterly Journal of Economics,* vol. 101, pp. 493-511.

[49] Zhou, J. (2011). 'Ordered search in differentiated markets', *International Journal of Industrial Organization,* vol. 29, pp. 253-262.