Diverging house prices

Jan Rouwendal¹ and Mark van Duijn
Department of Spatial Economics, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands
Corresponding author: j.rouwendal@vu.nl

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Abstract
This studies the well-known empirical phenomenon that houses of different quality experience different price developments. The typical pattern is that luxury houses appreciate more in boom periods and depreciate more during busts. The standard model of housing demand, developed by Muth, treats housing as a quantity of ‘housing services’, an imaginary homogeneous commodity that is available in arbitrary quantities at a constant price per unit. This model is unable to explain differential development of house prices. However, a simple variant that treats the number of houses offering a given number of housing services as fixed is able to do this. This is shown by means of a formal analysis of a model in which households that differ in income are allocated over a given housing stock. In particular, the model predicts that the price of housing as a function of quality becomes more convex after a proportional increase in all incomes. Earlier explanations of this phenomenon relied on down payment effects, but since diverging house price developments are also observed in countries where these effects are negligible, this provides only a partial explanation. We develop an empirical methodology for measuring house prices as a flexible function of housing services and use it to document the phenomenon of diverging house prices in Amsterdam in the period 1995-2011.

¹ Jan Rouwendal is also affiliated to the Tinbergen Institute and the Amsterdam School of Real Estate.
1 Introduction

Abundant evidence exists that prices of different types of housing evolve differently over time. Typically, luxury housing appreciates more than other types during booms, and depreciates more during busts. Figure 1 illustrates this phenomenon for detached and terraced housing in the Netherlands during a long period of house price increases that lasted from 1995 to 2007. The figures refer to existing owner-occupied houses (new construction is excluded) and are published by Statistics Netherlands and the Dutch Land Register (in Dutch: Kadaster). In all provinces, except Limburg, the prices of detached houses more than tripled. However, the prices of terraced houses never tripled but they doubled in all cases.

![Figure 1](Figure 1. Appreciation of detached and single family housing in Dutch provinces 1995-2007. Note: The figures show the ratio of prices in 2007 to 1995 minus 1, multiplied by 100. Source: Statistic Netherlands / Kadaster.)

The diverging house prices could be related to differences in the composition of subsets of sold houses over time. For instance, within the class of luxury houses very expensive houses may be sold much more frequently during boom periods. However, McMillen (2008) studied changes in the distribution of house prices in the Chicago metropolitan area and concluded that changes in the types of houses sold and their location are unable to explain the higher appreciation rates for

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3 Since 2007 house prices in the Netherlands have on average decreased.
high-priced homes. The phenomenon is therefore not a result of changes in the composition of
the set of sold houses, but it reflects genuine differences in the development of prices between
cheaper and more expensive segments of the housing market.

The differences in appreciation can easily be interpreted as the result of irrational (perhaps
speculative) behavior and they have important consequences for the homeowners. Moving to a
more luxury house becomes increasingly difficult during a boom period. The owners of such
luxury houses realize much larger (also in a relative sense) capital gains. As a consequence, the
user cost of luxury housing are relatively low during a boom period. Paradoxically, this increases
the attractiveness of luxury housing. During a bust period, the reverse of the above statements is
ture. The diverging house prices thus increase the volatility of the housing market and can easily
contribute to the impression of a developing bubble.

This impression can be reinforced by the inability of the conventional economic model of
housing demand to explain diverging house prices. This model is based on the concept of
‘housing services,’ an imaginary commodity introduced by Muth (1960) to facilitate the use of
standard micro-economic tools for housing market analysis. The idea is that house prices are in
fact the product of a constant unit price and a number of units of these housing services.
Expenditure on housing can therefore be decomposed into a price and a quantity component, just
like the expenditure of all other goods. The quantity is the amount of housing services that
determines the utility of the occupier of the house experiences. It is therefore natural to relate it to
the quality of the house as indicated by its characteristics. Moreover, the quantity of housing
services is assumed to be perfectly divisible and consumers can buy any desired amount of this
commodity at a constant unit price. The typical application of this approach thus considers the
housing stock as a large number of housing services that can be distributed arbitrarily over
households. Muth’s approach has been very successful and is the standard in economic analysis
of housing market. However, the constant unit price for housing implies that the price ratio of
houses that differ in quality must be constant over time, which excludes diverging house prices.

The explanations for diverging house prices that have been put forward in the literature
have therefore relaxed the housing services concept by distinguishing between two (or more)
types of houses which are imperfect substitutes and are not supplied completely elastically.

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4 This should be expected to have implications for the impact of changing house prices on consumption expenditure.
5 See, for instance, Rouwendal (1998) for an examination of the micro-economic foundations of the concept.
Ortalo-Magné and Rady (2006) show that in such a context a shock in income that leads to a modest increase in the price of low quality housing interact, via its impact on home equity, with the down payment constraint. This gives rise to a relatively large increase in the demand for high quality housing and thus to diverging price movements. However, the differential development of prices of different housing types can also be observed in countries such as the Netherlands where the down payment constraint hardly plays a role, as was shown in Figure 1.6

To explain diverging house price movements in a situation where the down payment constraint is absent, this paper develops a model that remains very close to the conventional housing services approach. The only modification we introduce is that we abandon the assumption of perfect malleability of housing services. That is, we assume that each house offers a given number of housing services in each period and that the number of houses producing a given number of housing services is fixed. There is thus not a single market for housing services, but a continuum of markets and for each type of housing (identified by the number of housing services it offers per period) there is a different price.

In this setup, the housing stock can be described as a distribution of houses that differ in quality. This stock is distributed by the market mechanism over a set of households. To focus on one important aspect – the relationship between income shocks causing housing market booms and differential price development – we assume that all households have identical tastes and differences in demand for housing thus depend only on differences in household income. We show that if housing is normal, the ranking of housing consumption follows that of incomes. This allows us to find the matching between incomes and houses. The requirement that this matching must be facilitated by the price mechanism implies a relationship between house price and quality, which can be viewed as a (one dimensional) hedonic price function.

The curvature of this price function is of some interest since it describes the relationship between price/quality ratio.7 Conditions under which the function is locally convex, linear or concave are made precise and they are shown to be related to the supply/demand ratio. Intuitively, if this ratio is large, the house price function must be locally convex to prevent demand from increasing ‘too fast’ with income. If the ratio is small, the house price function

6 In the Netherlands, a low priced mortgage insurance (the Nationale Hypotheek Garantie) is available for first time buyers. It allows them to finance the purchase of a house completely by a mortgage loan. To be eligible for the insurance the mortgage payment to income ratio should not exceed a threshold value of approximately 30%. However, this constraint does not have the same effects as a down payment constraint following an income shock.

7 Interpreted here as the ratio between the price of the house and the number of houses services produced by it.
must be concave to prevent a shortage. The situation in which the house price function is locally linear can be interpreted as an equilibrium in the sense that the number of available houses with a given quality matches the number of household demanding exactly at that quality.

Moreover, to show that the model explains stronger price increases of luxury housing in boom periods, we derive the conditions under which a proportional change in all incomes results in a more convex housing price function. The model is therefore able to explain the phenomenon of diverging house prices with a model that stays close to the Muth approach and without using acceleration effects related to binding down payment constraints. Intuitively our explanation is that when the income distribution stretches out by a proportional increase in all incomes, while the housing stock remains constant, market equilibrium requires that the increased demand for higher quality housing is dampened by price increases that are highest for the most luxury types of housing.

We then develop a methodology to estimate the housing price as a function of housing services. The method is based on the relationship between the ranking of houses based on prices and on housing services implied by our model. We use this property of the model to estimate an indicator of housing services. This allows us to study the development in the curvature of the housing price function over time. This methodology is applied to transaction prices in Amsterdam, the Netherlands. We find an almost continuous increase in convexity of the house price function during the prolonged boom period 1990-2007 and a decrease after the onset of the great depression.

The paper is organized as follows. The next section introduces the model and the main theoretical results. Section 3 discusses the implications of the model for the effect of income shocks on house prices and provides a simple example. Section 4 discusses the numerical solution of the general model and the incorporation of heterogeneity in tastes. Section 5 provides an empirical investigation of the convexity of the house price function in Amsterdam over the period 1995-2011. Section 6 summarizes and concludes.

2 The model

2.1 Introduction
We consider a market with a fixed supply of a heterogeneous commodity: housing. Houses are available in a continuum of varieties, and each variety is characterized by a number of housing services. This number can be regarded as a scalar index of housing quality. The consumers that demand housing have identical tastes, but differ in income. The housing stock is fixed in the short run.

Formally, we define housing quality as the number of housing services \( q \) offered by a house. The only departure from Muth’s (1960) framework is that we treat \( q \) as fixed for each house and allow the price per unit of housing services to differ over houses. The price (rent) \( p(q) \) of a house that offers \( q \) units of housing services per period is therefore not necessarily equal to the product of \( q \) and a unit price that is equal for all housing qualities.

The housing stock is described by the distribution function of the quality of housing, \( G(q) \). \( G \) is assumed to be differentiable and to have support on an interval \([q_{\min}, q_{\max}]\). The size of the stock of houses is \( S \), \( S = G(q_{\max}) \). The density function associated with \( G \) is denoted as \( g \).

The function \( p(q) \) can be interpreted as a simple hedonic price function. It gives the rent or user cost of a house as a function of its quality. We assume that the hedonic price function is twice differentiable. The marginal price of housing services, \( \pi \), is the first derivative of the hedonic price function:

\[
\pi = \frac{\partial p}{\partial q},
\]

Clearly, the marginal price of housing services is a constant if and only if the hedonic price function is linear, that is if: \( p(q) = \mu + \pi q \). In the familiar Muth case the house price is proportional to the number of housing services: \( \mu = 0 \). In the model we develop here, the hedonic price function will in general be nonlinear and does not have to pass through the origin.

The stock of houses is used by a population of households. Tastes are described by a utility function \( u \):

\[
u(q, c).
\]

The two arguments of this function are housing consumption \( q \), which is equal to the number of housing services offered by the house in which the household lives, and other consumption \( c \), which is summarized in the number of units of a composite good. The utility function is assumed

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8 The model we use can be regarded as a version of Sweeney’s commodity hierarchy model of the housing market with an infinite number of varieties. Braid (1981, 1984) used a similar setup to analyze rental housing markets.
to be two twice differentiable, increasing in its two arguments and to have convex indifference curves.

Consumers differ in income. The distribution of income is \( F^*(y) \), which has positive support on an interval \([y_{\min}, y_{\max}]\). We treat income as a continuous variable. Assume that \( F^* \) is differentiable and denote the density function as \( f \). The total number of households equals \( B \), where \( B = F^*(y_{\max}) \).

The budget constraint for each household is:
\[
c + p(q) = y.
\]
(3)

Maximization of the utility function (2) subject to condition (3) leads to the familiar first-order condition:
\[
\frac{\partial u}{\partial q} \frac{\partial u}{\partial c} = \frac{\partial p}{\partial q}.
\]
(4)

This condition implies that in the optimum the indifference curve touches the nonlinear budget line, as is illustrated in Figure 2.

Although we have emphasized that the hedonic price function is expected to be nonlinear, we make extensive use of the demand function, which is defined for a linear budget constraint. We denote the demand function as:
\[
q = q(\pi, y),
\]
(5)

where \( \pi \) denotes the – constant – marginal price for housing services.

With a nonlinear housing price function we can still describe consumer choice behavior in terms of the conventional demand function by linearizing the budget line at the optimum of the consumer. This implies that we use the marginal price of housing services, \( \partial p / \partial q \), in the optimum as the first argument of the demand function and virtual income, \( y^v \), which is defined as:
\[
y^v = y - p(q) + q \frac{\partial p}{\partial q},
\]
(6)
as its second argument. Demand function (5) is therefore rewritten as:
\[
q = q(\partial p / \partial q, y^v).
\]
(7)

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9 Since housing is durable, the appropriate income concept is permanent income.
In market equilibrium, each household must be on a demand function (7), with $y^v$ given as in (6). Note that the arguments of this demand function are determined by the choice the household makes on the housing market. That is, both the marginal price and virtual income are functions of the chosen amount of housing services $q$.

2.2 Two preliminary results
In this subsection we establish two elementary properties of the hedonic price function. The first one is that the hedonic price function is increasing in the number of housing services. To show this, suppose that the hedonic price function is not increasing in the number of housing services. Then there is at least one pair of housing services, say $q^1$ and $q^2$ with $q^2 > q^1$ and $p(q^2) < p(q^1)$. Since all consumers are utility maximizers, there will then be no demand for housing with quality...
The existence of such a pair is therefore incompatible with price equilibrium. Hence the user cost function must be increasing in the number of housing services.

The second result is that in a market equilibrium housing consumption must be increasing in income if housing is a normal good. This sounds somewhat trivial since normal goods are defined as goods whose consumption increases with income, but remember that this definition refers to a situation in which the unit price of the good is constant. It refers to the special case \( p(q) = \pi q \) only. What we will show now is that it also holds with a nonlinear hedonic price function. Fortunately, this is easy to show since the same logic applies with a nonlinear budget constraint. Housing is normal if and only if the marginal rate of substitution between housing and the composite commodity increases in the consumption of the composite commodity, that is if:

\[
\frac{\partial}{\partial c} \left( \frac{\partial u/\partial q}{\partial u/\partial c} \right) > 0. \tag{8}
\]

If the budget line shifts upward, its slope remains unchanged for any given level of housing consumption. This is true for a linear as well as a nonlinear budget line. However, the slope of the indifference curve through the point of the budget line corresponding to this given level of housing consumption gets steeper, if inequality (7) holds. This implies that the optimal level of housing consumption must be larger after the vertical shift of the budget line than it was before. The same reasoning applies to a downward shift.

This is illustrated in Figure 3. In this figure two budget lines are drawn as \( q = y - p(q) \) for a nonlinear hedonic price function. The lowest budget line touches the indifference curve \( ic^1 \). For given housing consumption, for instance \( q^1 \), the slopes of the two budget lines are equal. If the slopes of the indifference curves crossing or touching the two budget lines would also be equal, housing could not be a normal good.\(^{10}\) Indifference curve \( ic^2 \) must therefore be steeper than \( ic^1 \) when housing consumption equals \( q^1 \) and optimal housing consumption at the higher income level must exceed \( q^1 \). This second result ensures that, in equilibrium, a household’s position in the income distribution is reflected in its position in the housing stock, even if the housing price function is nonlinear.

\(^{10}\) Note that for this conclusion the nonlinearity of the budget constraint does not matter.
Earlier in this section we introduced the income and housing distributions. The result just reached implies that there is an intimate relationship between the two distributions. Consider the situation in which the number of households is at least as large as the housing stock: \( B \geq S \). Then only the households with the highest income will be able to live in a house. The remaining households \((B - S)\) can be interpreted as potential households, which will only be formed if the situation on the housing market permits. Alternatively, the housing stock \( S \) may refer to a part of

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**Figure 3.** Normal goods and a nonlinear budget line
the housing market only, for instance owner-occupied houses. We will use the latter example in what follows.

Let \( y^c \) be the lowest income of households with an owner-occupied house. The results just derived imply that the household with this income lives in the house of the lowest quality \( q^\text{min} \) and pays the lowest price \( p(q^\text{min}) \). Similarly, the household with the highest income \( y^\text{max} \) lives in the house with the highest quality and pays the highest price for housing. More generally, we can order the incomes of the homeowners from low to high and we can similarly order the quality of the houses from low to high. The order of the incomes must be the same as the order of the housing qualities. We can therefore determine the pairs of incomes and housing qualities that must match. We denote the income \( y \) that is associated with housing quality \( q \) as \( y(q) \).

The relationship between income and housing consumption implies:

\[
F^*(y(q)) - F^*(y^c) = G(q),
\]  
which follows from our earlier result that housing consumption is increasing in income. We use the more convenient notation \( F(y) = F^*(y) - F^*(y^c) \) for the part of the income distribution that refers to households with positive housing consumption. Using this notation, we can rewrite (9) as:

\[
y(q) = F^{-1}(G(q)).
\]  
This gives the relationship between income and housing consumption in this model. Note that it could be determined on the basis of some general properties of the allocation process and that the role of prices is not yet made explicit.

For later reference, we note that (10) implies:

\[
\frac{dy(q)}{dq} = \frac{g(q)}{f(y(q))},
\]

where \( g \) and \( f \) are the densities associated with the distributions \( G \) and \( F \), respectively.

### 2.3 Market equilibrium and the curvature of the hedonic price function

In a market equilibrium each household must be on demand curve (7) and the implied combination of income and housing consumption should satisfy (10). That is, in market equilibrium we can rewrite (7) as:

\[
q^*(y) = q \left( \frac{\partial p}{\partial q} \cdot y(q) - p(q) + \frac{\partial p}{\partial q} q \right).
\]  
Substitution of (10) into (11) gives:
\[ q^*(y) = q \left( \frac{\partial p}{\partial q}, F^{-1}(G(q)) - p(q) + \frac{\partial p}{\partial q} q \right). \]  (12)

This equation defines the market equilibrium in the model.

We will now characterize the nonlinearity of the hedonic price function. In order to do that we focus on its second derivative, which gives the change in the marginal price of housing. With a linear hedonic price function, this second derivative equals 0, but in general it is nonzero, as we show in proposition 1.

**Proposition 1** In market equilibrium the second derivative of the hedonic price function is:

\[
\frac{\partial^2 p}{\partial q^2} = \frac{\frac{\partial q}{\partial y} g(q)}{\frac{\partial f(y(q))}{\partial y} + \frac{q}{\partial y}} - 1,
\]  (13)

with \( \pi = \frac{\partial p(q)}{\partial q} \), the marginal price of housing services.

To show this, we differentiate the equilibrium demand equation (12) with respect to \( q \). The result is:

\[
dq = \frac{\partial q}{\partial y} \left( \frac{g(q)}{f(y(q))} dq + \left( - \frac{\partial p}{\partial q} + \frac{\partial p}{\partial q} + q \frac{\partial^2 p}{\partial q^2} \right) dq \right) + \frac{\partial q}{\partial y} \frac{\partial^2 p}{\partial q^2} dq.
\]  (14)

After removing the terms that cancel and rearranging the remaining terms, this gives:

\[
1 - \frac{\partial q}{\partial y} \frac{g(q)}{f(y(q))} = \left( \frac{\partial q}{\partial \pi} + \frac{q}{\partial y} \right) \frac{\partial^2 p}{\partial q^2}.
\]  (15)

Solving this equation for \( \frac{\partial^2 p}{\partial q^2} \) gives (13).

To interpret (13), observe that the expression between parentheses in the denominator is the Slutsky term of the demand equation for housing. It is negative if the demand for housing is consistent with utility theory. Assuming this condition is satisfied, we conclude that the proposition says that the hedonic price function is linear when:

\[
\frac{\partial G}{\partial q} \frac{\partial q}{\partial y} = \frac{\partial F}{\partial y}.
\]  (16)

This can be interpreted as a local equilibrium condition that holds when the housing stock and the income distribution are balanced: the density of households with a particular income level \( y \) is matched perfectly with the density of houses that have the quality level \( q \) demanded by these households at the prevailing marginal price of housing.

The hedonic price function is (strictly) convex when
\[
\frac{\partial G}{\partial q} \frac{\partial q}{\partial y} > \frac{\partial F}{\partial y},
\]
and strictly concave when:
\[
\frac{\partial G}{\partial q} \frac{\partial q}{\partial y} < \frac{\partial F}{\partial y}.
\]

To show what this means, observe that the densities on left-hand sides of equations (17) and (18) give numbers of houses and the densities on the right-hand side numbers of households. The slopes of the Engel curve, which also appear on the left-hand sides, translate the number of houses into corresponding numbers of households. The houses of which the number is indicated on the left are those demanded by the households whose number is indicated on the right. If the translation of houses into households results in equal numbers on both sides of the equation, the hedonic price function is linear. If this is not the case, the hedonic price function must be nonlinear in order to match all households to houses.

If (17) holds, there are more houses available than needed for the households to be on their demand curve if the marginal price is fixed. Equilibrium can therefore only be realized in this part of the stock when the marginal price changes. More precisely, the marginal price must increase in order to slow down the increase of demand with income so that all houses in this part of the stock will be demanded. Hence, equation (13) implies that \( \partial^2 p / \partial q^2 > 0 \) in this case. In the alternative case, when (18) holds, analogous reasoning shows that the hedonic price function is concave.

Since we have assumed that housing is a normal good, and \( \partial y(q) / \partial q \) is nonnegative, the value of the numerator on the right hand side of (12) has -1 as its lower bound. This implies that there is also a bound on the possible concavity of the hedonic price function (i.e. on the absolute value of \( \partial^2 p / \partial q^2 \) whenever it is negative), whereas there is no such upper bound on the convexity. To see what the upper bound on the concavity implies, we consider the Hicksian demand curve for housing \( q^H = q(\pi, u) \). If we move along this demand curve, we have: \( dq = (\partial q^H / \partial \pi) d\pi \) or \( d\pi / dq = 1 / (\partial q^H / \partial \pi) \). Now observe that \( \pi \) is the slope of the indifference curve corresponding to the Hicksian demand, and that \( d\pi / dq \) is the second derivative of this indifference curve. This second derivative equals \( 1 / (\partial q^H / \partial \pi) \), which is minus the upper bound of \( \partial^2 p / \partial q^2 \). We therefore conclude that the concavity of the hedonic price function is bounded by the convexity of the indifference curve. That is, \( -p(q) \) cannot be more convex than the indifference curve to which it is tangent. This is illustrated in Figure 4.
Figure 4. A locally concave hedonic price function for housing

Figure 4 shows a non-linear budget line, which is partly convex, because the hedonic price function is partly concave. However, in the optimum, the convexity of the budget line is less than that of the indifference curves. The highest indifference curve that can be reached touches the budget line: the two have just a single point in common. The budget line is less convex than the indifference curve.

3 Income shocks and house prices

3.1 General discussion
The model of the present paper is a static one. A constant distribution of incomes can, however, be the result of a dynamic underlying development in which starters enter the housing market with low incomes, gradually realize income increases and finally leave the model. In the course of their lifetime individual households change from modes to more luxury housing, climbing up the quality ladder. The macro picture is then a static one, although the underlying process is dynamic at the household level. For each level of housing quality there may be households who
want to sell, because their income has gone up or down or perhaps because they want to leave the market. There may also be households who want to buy, because their income has changed or because they want to enter the market. As long as the distributions of income and housing remain unchanged, demand and supply will be equal on each submarket at the prevailing housing price function.

It is interesting then to see what happens if a shock occurs to this underlying process. For instance, (lifetime) income perspectives may change which results in a change in the distribution of income. Consider, for instance, a change in the income distribution from $F^0(y)$ to $F^1(y)$, while the total number of households and the housing stock remain unchanged. For concreteness we assume that the income distribution shifts to the right, that is $F^0(y) \leq F^1(y)$ for all incomes, and the inequality should be strong for some incomes. This means that for some – relatively low – incomes demand decreases as $\frac{\partial F}{\partial y}$ becomes smaller, whereas for other – higher – incomes $\frac{\partial F}{\partial y}$ may get larger. The discussion of the previous section suggests that the former change will result in a more convex housing price function, whereas the latter results in a more concave housing price function. The first change implies that higher quality housing will appreciate more than low quality housing – hence housing price diverge – but the second effect may counteract this. The fact that the income distribution shifts to the right suggests that the former impact will be stronger than the latter, as the desire to trade up in general becomes stronger, but without further investigation we cannot be sure if this is indeed the case.

We consider two specific shifts in the income distribution: first an identical absolute change in all incomes, then an identical proportional change. In the first case we have $F^1(y + \Delta) = F^0(y)$, where $\Delta$ denotes the common change in income and we have used super fixes to distinguish the two density functions. The matching of households to houses requires that households with income $y + \Delta$ now inhabit houses formerly used by households with income $y$. Since $f^1(y + \Delta) = f^0(y)$ the ratio $\frac{\theta(q)}{f(y(q))}$ in (13) remains unchanged for all $q$. The curvature of the hedonic price function may nevertheless change when the higher income (at a given value of $q$) affects the slopes of the Engel curve or the demand curve (or both). Although general statements cannot be made, it seems likely that the absolute value of the Slutsky term will decrease, which would imply more curvature of the hedonic price function: if it was concave it becomes more concave, if it was convex it becomes more convex. If the slope of the Engel curve
also decreases, this would strengthen the impact on concavity, and counteract the impact on convexity.

Now consider the situation in which all incomes increase with the same percentage: \( F^1(ky) = F^0(y) \) for some \( k > 1 \). Matching of the households to the housing stock now requires that households with income \( ky \) occupy the houses formerly inhabited by households with income \( y \). Moreover, we must have \( f^1(ky) = \frac{f^0(y)}{k} < f^0(y) \), which tells us that the term \( \frac{g(q)}{f(y(q))} \) in (13) now increases. This makes the hedonic price function more convex in the sense that it increases the value of the second derivative of this function.

The slope of the Engel curve and the Slutsky term may also change in this case, and this complicates the picture of course. It seems likely that the absolute value of the Slutsky term decreases when income changes and this will cause no problems. However, the slope of the Engel curve may decrease and since this counteracts the movement towards a more convex price function we look at it in some detail. It may be noticed that the net change in the numerator of the right-hand side of (13) remains positive after all incomes increase with a factor \( k > 1 \) if \( \frac{\partial q(ky)}{\partial y} > \frac{1}{k} \frac{\partial q(y)}{\partial y} \). A sufficient condition for this inequality to hold is \( \frac{\partial^2 q(ky)}{\partial y^2} > -\frac{\partial q(y)}{\partial y} \frac{1}{yk^2} \). This shows that some concavity of the Engel curve for housing is compatible with a price function that becomes more convex after an income shock. It is not difficult to verify that the linear and loglinear Engel curves satisfy this criterion.12 Concluding, we may state:

**Proposition 2** If the absolute value of the Slutsky term is non-increasing in income and the Engel curve for housing services is not too concave in the sense that \( \frac{\partial q(ky)}{\partial y} > \frac{1}{k} \frac{\partial q(y)}{\partial y} \) for all \( k > 1 \), then a proportional increase in all incomes causes the second derivative of the house price function to increase everywhere.

### 3.2 A linear example

11 Define \( A(k) = \frac{\partial q(ky)}{\partial y} > \frac{1}{k} \frac{\partial q(y)}{\partial y} \). Clearly, \( A(0) = 0 \) and the condition \( \frac{\partial q(ky)}{\partial y} > \frac{1}{k} \frac{\partial q(y)}{\partial y} \) holds if \( \frac{\partial A(k)}{\partial k} > 0 \) for all \( k > 0 \). Since \( \frac{\partial A(k)}{\partial k} = \frac{\partial^2 q(ky)}{\partial y^2} y + \frac{1}{k^2} \frac{\partial q(y)}{\partial y} \) which is positive if \( \frac{\partial^2 q(ky)}{\partial y^2} > -\frac{1}{yk^2} \frac{\partial q(y)}{\partial y} \).

12 Remember that we have assumed housing to be normal.
To illustrate the model further, we consider an example. Assume that preferences are such that the demand function for housing is linear:

\[ q = a + b\pi + cy, \]  

(19)

and that the distributions of income and housing stock are uniform:

\[ F(y) = \frac{y}{y_{\text{max}} - y_c}, \]  

(20)

\[ G(q) = \frac{q}{q_{\text{max}} - q_{\text{min}}}. \]  

(21)

The maximum income should be small enough to keep the Slutsky term of the linear demand equation (19) negative, as is required by economic theory. Equation (13) implies:

\[ \frac{\partial^2 p}{\partial q^2} = \frac{c y_{\text{max}} - y_c q_{\text{max}} - q_{\text{min}} - 1}{-(b+qc)}. \]  

(22)

Differential equation (22) can be solved as:

\[ p(q) = p(q_{\text{min}}) + \left( \left( C + \pi(q_{\text{min}}) \right) - \frac{1}{c} \right) (q - q_{\text{min}}) + \frac{1}{c^2} (1 - cC)(b + cq) \ln \left( \frac{b+qc}{b+qc_{\text{min}}} \right), \]  

(23)

where \( C = \frac{y_{\text{max}} - y_c}{q_{\text{max}} - q_{\text{min}}} \). It is clear from (23) that the second derivative of the hedonic price function equals 0 if \( cC = 1 \), and in that case (23) simplifies to:

\[ p(q) = p(q_{\text{min}}) + \pi(q_{\text{min}})(q - q_{\text{min}}). \]  

(24)

We can compute the value of \( \pi(q_{\text{min}}) \) from the requirement that the owner-occupying household with the lowest income chooses the house with the lowest quality:

\[ q_{\text{min}} = a + b\pi(q_{\text{min}}) + cy_{\text{min}}. \]  

(25)

This gives \( \pi(q_{\text{min}}) = (q_{\text{min}} - a - cy_{\text{min}})/b \). The value of \( p(q_{\text{min}}) \) is determined by the requirement that the owner-occupying household with the lowest income should be able to reach the same level of utility in rental housing.
The linear hedonic is, of course, a special case. If \( cC > 1 \) the coefficient for \( (q - q^{min}) \) in the second term of \( p(q) \) is a constant that is larger than \( \pi(q^{min}) \), and the third term is non-zero. If \( cC > 1 \) this third term is negative and convex. If \( cC < 1 \) the coefficient for \( (q - q^{min}) \) is smaller than \( p(q) \) and the third term is positive and concave.

A simple numerical example can be constructed as follows. The parameters of the demand function are chosen as: \( a=13, b=-2, c=0.01 \). Incomes are between \( y^{min}=10 \) and \( y^{max}=100 \). This implies that the Slutsky term \( b + cy \) varies between -1.9 and -1. Housing quality varies between \( q^{min}=1 \) and \( q^{max}=10 \). The market is equilibrated by a linear hedonic price function that passes through the origin. The price per unit of housing services equals 6.5.

If all incomes increase with 1 unit, the market is equilibrated by a unit price of 6.55 for housing services. This requires that the price of the owner-occupied house of minimum quality now also has a price of 6.55. This might be due to an increase in rent that parallels the increase in user costs. If rents remain unchanged and the price of the lowest quality owner-occupied house is constant at 6.5, the new marginal price of housing is slightly higher: 6.5526. The hedonic price function is still a straight line, but it does not pass through the origin.

If all incomes increase by 5\%, the hedonic price function is no longer linear. The marginal price increases from 6.525 for \( q=q^{min} \) to 6.846 for \( q=q^{max} \) when it is assumed that the user cost of
the smallest owner occupied house also increases to 6.525. Again, results are slightly different when the price of this house is kept constant. The results for decreases in incomes are, of course, similar but in the opposite direction.

Figure 5 illustrates the model for the 20% changes in income and all other parameters identical to those we just discussed. The upper panel shows the hedonic price functions in the original situation (in which it is linear) and with the higher and lower incomes, whereas the lower panel pictures the marginal prices in each of the three situations.

The just shown results for a specific case can be generalized to arbitrary linear demand curves. First consider a change in the income distribution by which all incomes grow with the same absolute number $\Delta y$. The income change implies that the demand for housing quality of each household increases with $c \Delta y$. This implies that demand for the lowest quality houses disappears completely, while there is now demand for houses of a somewhat higher quality than the maximum currently available in the market. The old equilibrium thus no longer holds. To find the new one, note first that $y^{\text{max}}$ and $y^c$ both increase by $\Delta y$, which implies that $C$ will not change. This tells us that if the hedonic price function were linear in the original situation, it will again be so in the new equilibrium. Also if it were convex or concave, this will not change.

Assuming a linear hedonic price function in the original situation, we know that the new equilibrium price $\pi^{**}$ must satisfy $q^{\text{min}} = a + b\pi^{**} + c(y^c + \Delta y)$. From this it is easy to compute that $\pi^{**} = \pi^* + (c/b)\Delta y$, where $\pi^*$ denotes the original equilibrium price. This means that the prices of all housing qualities increase proportional to their quality. In other words, incomes change by the same number but house prices with the same percentage. Note also that in this example all households remain in the same house. All that changes is that a higher price has to be paid for these houses. And there is, of course, a wealth effect for the owners of the houses.

Now consider the effect of a proportional change in all incomes: all incomes change by the same percentage. This means that the difference between $y^{\text{max}}$ and $y^c$ increases and therefore the value of $C$ changes. If the hedonic price function is linear in the original situation, it will be convex in the new one when incomes increase and concave when incomes decrease. Proportional changes in incomes will therefore lead to changes in house prices that are not proportional to quality. The relative change in the housing price will be largest for the highest quality houses. This will probably stimulate the supply of high quality houses.
4 Further discussion
In this section we briefly discuss two other issues. Can we solve the model – i.e. derive the hedonic price function – when the assumptions of the linear case discussed in 3.2 do not hold? And, what will change if households do not only differ in income but also in preferences.

4.1 Solving the model in the general case
To see how the model can be used with an arbitrary demand curve, return to (11), which we repeat here:

\[ q^*(y) = q \left( \frac{\partial p}{\partial q}, y(q) - p(q) + \frac{\partial p}{\partial q} q \right). \] (26)

We assume that the distributions of income and housing and the indirect utility function associated with the demand curve are known. This allows us to find the matching function \( y(q) \) and therefore the income that corresponds to the housing of minimum quality: \( y^* = y(q_{\text{min}}) \). At this minimum income a household must be indifferent between the owner occupied housing of minimum quality and a substitute, for instance rental housing. This condition allows us to determine the price of the lowest quality housing \( p(q_{\text{min}}) \). Also imposing the condition that this household is on its demand curve then gives us the marginal price \( \pi(q_{\text{min}}) \) from (26). This brings us in a position in which we can use standard methods for solving differential equations, for instance Euler’s method, to trace out the complete hedonic price function \( p(q) \).13

4.2 Heterogeneity in preferences
Until now we have only considered heterogeneity in incomes. To deal with a situation in which actors can also differ in tastes, we now generalize the model to a situation in which the utility function is \( u(q, c; \varepsilon) \), where \( \varepsilon \) is a possible vector valued variable that indicates taste heterogeneity. We assume a simultaneous density function \( f^*(y, \varepsilon) \). Demand for housing can be written as \( q = q(y^v, \pi, \varepsilon) \). The consumer is a homeowner when the maximum utility of owning exceeds that of renting and we denote the set of combinations \((y, \varepsilon)\) for which this is the case with a given hedonic price function as \( O(p(q)) \).

13 See, for instance, Judd (1998).
The distribution of the demand for housing at a given hedonic price function will be denoted as \( H(q; p(q)) \). It is defined as:

\[
H(q; p(q)) = \iint_{(y,\epsilon)\in O(p(q))} f^*(y, \epsilon) dy d\epsilon.
\]

(27)

The distribution of houses is denoted as before as \( G(q) \). A price equilibrium is a housing price function \( p(q) \) for which:

\[
H(q; p(q)) = G(q) \text{ for all } q \in [q_{\text{min}}, q_{\text{max}}].
\]

(28)

This implies:

\[
h(q; p(q)) = g(q) \text{ for almost}^{14} \text{ all } q \in [q_{\text{min}}, q_{\text{max}}].
\]

(29)

where \( h(.) = \partial H / \partial q \). A given demand for housing services \( q \) can be generated by different combinations of \( y \) and \( \epsilon \) and we can write the income that generates \( q \) as a function of \( \epsilon \) by inverting the demand function:

\[
y = p(q) - \pi q + z(\epsilon, \pi; q).
\]

(30)

Using this, we can write:

\[
h(q; p(q)) = \int_{(y,\epsilon)\in O(p(q))} f^*(p(q) - \pi q + z(\epsilon, \pi; q), \epsilon) d\epsilon.
\]

(31)

This can be used to find an expression for \( h(.) \) from a demand function and the simultaneous distribution of income and the taste heterogeneity parameter. Numerical techniques can then be used to find the equilibrium price function.

To provide an example, consider the case of a linear demand function in which taste heterogeneity is introduced by means of a random intercept:

\[
q = a + \epsilon + b\pi + cy.
\]

(32)

This allows one to summarize all heterogeneity in a scalar \( \mu = y + \frac{1}{a} \epsilon \). The distribution of \( \mu \) can be derived from the simultaneous density \( f^*(y, \epsilon) \), and then one can proceed as in the example given above. This – admittedly special – case is not more difficult to handle as the corresponding one in which there is no taste heterogeneity. However, note that also in this simple case, taste heterogeneity implies that there is no longer a strict one-to-one relationship between income and housing consumption.

---

\(^{14}\) Except for a set of measure zero.
5 Diverging house prices in Amsterdam

In this section we study the development of the house price function $p(q)$ in the period 1995-2011 in Amsterdam, the largest Dutch city. During the first 12 years the housing market was booming, especially in the late 1990s. Since 2008 the impact of the global crises was felt. Positive income shocks are expected to result in increasing average house prices as well as increasing convexity of the house price function, whereas the opposite holds for negative shocks.

We use transaction data provided by the Dutch association of realtors, abbreviated (in Dutch) as NVM. They contain information on transaction prices and housing characteristics of houses sold by members of this association. Summary statistics of our data are provided in Table 1.

During the period 1995-2007 the Amsterdam housing stock was not completely constant, but growth was slow: on average 0.66% annually. The total size of the housing stock was just over 350,000 houses in 1995 and 380,000 in 2007. The growth in the number of owner-occupied housing was somewhat larger than that of the total: it increased from 38,000 to 84,888. The reason is that housing association sold part of their property in the period under consideration. Unfortunately, we have no detailed information about the quality of these additions to the stock of owner-occupied housing. New houses and houses sold by housing associations are underrepresented in our data as member of the NVM concentrate on the sale of existing housing. However, it is well known that since the early 1990s Amsterdam has tried to attract – or keep – high income households by supplying more housing in the owner-occupied segment. There is also a general impression that housing associations have predominantly sold houses of the highest quality they owned. If anything, this suggests that the additional supply of owner occupied housing was somewhat overrepresented in the luxury segment and should therefore be expected to have counteracted any tendency towards more convexity of the housing price function.

The slow growth of the housing stock was accompanied by growth of the household population. Unfortunately, we do not have detailed information about the income distribution of Amsterdam households and its development over the period under study. It is known, however, that the municipality tried to attract more higher income households to the city and this was, indeed, a main background of the emphasis on more owner-occupied housing. It is also well known that higher educated households, which often also have higher incomes, are especially attracted by urban amenities. Our interpretation is therefore that the gradual increase in income
over the period and the inflow of higher income households have compensated the impact of the increase in housing quality that was due to the construction of new owner-occupied housing. In other words, we expect the impact of the gradual increase in income over the period to be the dominating force on the housing price function which is therefore expected the have become more convex in the course of time.

5.1 Estimation strategy
A major difficulty in applying the model developed in the previous section is that we cannot observe housing services. To construct a measure of this crucial variable, we exploit a property of the model developed above: the ranking of houses on the basis of housing services is identical to that on the basis of prices. This ranking therefore reveals information about the housing services that we will exploit that to estimate a measure of housing services. Once we have this measure, we can compare the development of the housing price function \( p(q) \) over time.

We assume that housing services are a function of observed and unobserved housing characteristics:

\[
q = q(h) + \xi. \quad (33)
\]

In this equation \( h \) is a vector of observed housing characteristics, and \( \xi \) is a random variable that reflects the unobserved characteristics. As we noted, an elementary property of our model is that in each market and in each period the house prices is a monotone increasing function of the number of housing services provided by the house. This implies that the ranking of houses on the basis of price reflects the ranking on the basis of the number of housing services, although the strict proportionality of the Muth model is lost. We thus have:

\[
p_i > p_j \iff q(h_i) + \xi_i > q(h_j) + \xi_j, \quad (34)
\]

where the suffixes \( i \) and \( j \) denote arbitrary houses observed on the same market and in the same period.

To be able to estimate the function \( q \) that links housing characteristics to housing services, we assume that \( \xi_j \) is Extreme Value type I distributed and apply the results of Beggs, Cardell and Hausman (1981) on the ranking of a set of choice alternatives. We have observations on prices and housing characteristics for a number of years \( t=1..T \) and we order the observations within each year on the basis of their prices: the most expensive house in year \( t \) is indexed 1,\( t \), et cetera.
The likelihood of observing the actual ranking of these houses on the basis of the prices in year \( t \) is then given as:

\[
L_t = \frac{e^{q_{1,t}}}{\sum_{i=1} e^{q_{i,t}}} \frac{e^{q_{2,t}}}{\sum_{i=2} e^{q_{i,t}}} \frac{e^{q_{3,t}}}{\sum_{i=3} e^{q_{i,t}}} \cdots \frac{e^{q_{n(t)-1,t}}}{\sum_{i=n(t)-1} e^{q_{i,t}}} \frac{e^{q_{n(t),t}}}{\sum_{i=n(t)} e^{q_{i,t}}},
\]

(35)

where \( q_{i,t} \) denotes \( q(h_i) \) for the house ranked \( i \) in year \( t \), and \( n(t) \) denotes the total number of observations in year \( t \). We pool the observations for all years and maximize the likelihood of all observations:

\[
L = \prod_t L_t. \tag{36}
\]

This means that we use the same specification of the housing services function \( q(. \) in all periods. Moreover, we specify \( q(\cdot) \) as being linear in the parameters to be estimated:

\[
q(h) = \sum_{k=1}^{K} \beta_k h_k. \tag{37}
\]

This specification of housing quality is consistent with what is often used in hedonic price equations.

It is important to note the difference between (37) and the conventional linear-in-parameters specification of a hedonic price function. Although the same variables appear on the right-hand side of (34) they refer now to the ranking of the house, not to its price. If differences in ranking correspond closely to differences in price, for instance because prices of sold houses are uniformly distributed in some price interval, the parameters in (37) could be close to those of a hedonic regression, but there is no reason why prices should be uniformly distributed. Explaining the rank order of houses on the basis of prices is therefore different from explaining the prices themselves. Needless to say, the estimated coefficients of (37) cannot be interpreted as reflecting (average) willingness to pay.

It can be argued that the model we have just discussed is somewhat more restrictive than is necessary, because the attractiveness of neighborhoods, which is part of the housing services, can change over time due to changes in household composition, shopping possibilities et cetera. We have therefore estimated two variants of the model: one in which all coefficients are assumed to be constant over time, and a second in which we allow the coefficients for neighborhood dummies to be year-specific.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transactions (in euros)</td>
<td>254,975</td>
<td>170,158</td>
<td>25,900</td>
<td>1,500,000</td>
</tr>
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<td>43.38</td>
<td>10</td>
<td>919</td>
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<td>Rooms (#)</td>
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<td>1</td>
<td>10</td>
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<td>11.66</td>
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<td>0.09</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
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<td>0.15</td>
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<td>1</td>
</tr>
<tr>
<td>Semidetached house (ref: standard house)</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
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<tr>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Terrace</td>
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<td>0.31</td>
<td>0</td>
<td>1</td>
</tr>
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<td>Bad inside maintenance</td>
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<tr>
<td>Monument</td>
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<td>0</td>
<td>1</td>
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<tr>
<td>1 Centrum</td>
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<td>0.36</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2 Slotervaart en Overtoomse Veld</td>
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<td>0.21</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3 Zuidoost</td>
<td>0.06</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4 Oost en Watergraafsmeer</td>
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<td>0.24</td>
<td>0</td>
<td>1</td>
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<td>5 Amsterdam Oud-Zuid</td>
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<td>0.20</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6 Zuideramstel</td>
<td>0.05</td>
<td>0.22</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7 Westerpark</td>
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<td>0</td>
<td>1</td>
</tr>
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<td>8 Oud-West</td>
<td>0.03</td>
<td>0.18</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9 Zeeburg</td>
<td>0.05</td>
<td>0.22</td>
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<td>1</td>
</tr>
<tr>
<td>10 Bos en Lommer</td>
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<td>0.20</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>11 De Baarsjes</td>
<td>0.06</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>12 Amsterdam-Noord</td>
<td>0.07</td>
<td>0.26</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>13 Geuzenveld en Slotermeer</td>
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<td>0.37</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>14 Osdorp</td>
<td>0.09</td>
<td>0.28</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Once we have a measure of housing services, we can proceed to estimate the housing price function $p(q)$. A difficulty that emerges is that the proper argument of this function is $\sum_{k=1}^{K} \beta_k h_k + \xi$, while we do not have information about the last term, $\xi$. Since the price function is in general nonlinear, it is not of much help that we can assume that the expected value of $\xi$ equals 0. However, it helps to assume that the median of this variable equals 0, because the median of $p(\sum_{k=1}^{K} \beta_k h_k + \xi)$ equals $p(\sum_{k=1}^{K} \beta_k h_k)$. This suggests the use of quantile (median) regression to estimate $p(q)$.

As noted before, the housing price function $p(q(h))$ can be interpreted as a hedonic price function. It is a ‘single index’ function in which the changes in the marginal prices of all relevant housing characteristics are closely related to each other.

5.2 Results: Housing services and housing characteristics

Estimation results for the housing services function $h(q)$ are reported in Table 2. The annual number of available observations increased gradually over the years. In order to keep estimation tractable, we imposed a maximum of 2,000 on the number of observations to be used per year. If the number of available observations was larger, we have randomly drawn 2,000 observations from the complete set. All coefficients for housing characteristics have the expected sign and most of them are highly significant.

The estimation results for the neighborhood dummies confirm expectations based on prior knowledge. When we allow the coefficients of these neighborhood dummies to differ over the years, we then estimated coefficients of the housing characteristics change modestly. We used the center as the reference in each period.
### Table 2. Estimation results for housing services (1995 - 2011)

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th></th>
<th>(2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor space (m²)</td>
<td>0.020 (0.0002) ***</td>
<td>0.021 (0.0002) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rooms (#)</td>
<td>0.298 (0.0061) ***</td>
<td>0.311 (0.006) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance to city center (km)</td>
<td>-0.042 (0.0067) ***</td>
<td>-0.047 (0.0067) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detached house (ref: standard house)</td>
<td>-0.043 (0.0572)</td>
<td>1.033 (0.0656) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corner house (ref: standard house)</td>
<td>0.317 (0.0395) ***</td>
<td>0.382 (0.0398) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semidetached house (ref: standard house)</td>
<td>0.946 (0.065) ***</td>
<td>0.909 (0.0676) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartment (ref: standard house)</td>
<td>-0.200 (0.0224) ***</td>
<td>-0.152 (0.0226) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcony</td>
<td>0.024 (0.012) **</td>
<td>0.033 (0.0121) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dormer</td>
<td>-0.117 (0.0377) ***</td>
<td>0.002 (0.0398)</td>
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<td></td>
</tr>
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<td>Terrace</td>
<td>0.477 (0.0197) ***</td>
<td>0.336 (0.0198) ***</td>
<td></td>
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</tr>
<tr>
<td>Park</td>
<td>0.546 (0.0221) ***</td>
<td>0.587 (0.0226) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Garden</td>
<td>0.831 (0.0466) ***</td>
<td>0.801 (0.0462) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well-maintained garden</td>
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<td>0.618 (0.0217) ***</td>
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<td>Bad inside maintenance</td>
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<td>-0.667 (0.0186) ***</td>
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<tr>
<td>Bad outside maintenance</td>
<td>-0.663 (0.0271) ***</td>
<td>-0.552 (0.0272) ***</td>
<td></td>
<td></td>
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<td>Monument</td>
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<td>0.340 (0.0306) ***</td>
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<td>Neighborhood dummies</td>
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<td>-</td>
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<tr>
<td>Neighborhood * Year dummies</td>
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<tr>
<td>Log likelihood</td>
<td>-204,039</td>
<td>-202,720</td>
<td></td>
<td></td>
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<tr>
<td>Observations</td>
<td>34,351</td>
<td>34,351</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are in parentheses. Significance at 90, 95 and 99% level are, respectively, indicated as *, **, ***. The reference neighborhood is 1 Centrum. The other coefficients can be obtained by the author.
Table 3. Quadratic specification of the housing price function

<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Housing services</td>
<td>11,580***</td>
<td>26,263***</td>
<td>17,858***</td>
<td>16,178***</td>
<td>15,883***</td>
<td>13,062***</td>
<td>7,765***</td>
<td>17,040***</td>
<td>18,117***</td>
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<td></td>
<td>(1,393)</td>
<td>(1,796)</td>
<td>(1,132)</td>
<td>(1,276)</td>
<td>(1,522)</td>
<td>(2,435)</td>
<td>(2,400)</td>
<td>(2,151)</td>
<td>(2,780)</td>
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<tr>
<td>(Housing services)^2</td>
<td>3,816***</td>
<td>1,923***</td>
<td>4,226***</td>
<td>6,197***</td>
<td>8,200***</td>
<td>10,698***</td>
<td>11,553***</td>
<td>9,202***</td>
<td>7,820***</td>
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<td></td>
<td>(209.4)</td>
<td>(257.2)</td>
<td>(182.2)</td>
<td>(214.2)</td>
<td>(246.6)</td>
<td>(434.3)</td>
<td>(383.1)</td>
<td>(371.2)</td>
<td>(422.7)</td>
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<tr>
<td>Constant</td>
<td>34,885***</td>
<td>25,450***</td>
<td>54,601***</td>
<td>69,827***</td>
<td>95,057***</td>
<td>113,187***</td>
<td>130,607***</td>
<td>131,513***</td>
<td>110,810***</td>
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<tr>
<td></td>
<td>(2,103)</td>
<td>(2,777)</td>
<td>(1,550)</td>
<td>(1,659)</td>
<td>(2,019)</td>
<td>(3,040)</td>
<td>(3,299)</td>
<td>(2,715)</td>
<td>(4,049)</td>
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<td>3,007</td>
<td>3,354</td>
<td>3,807</td>
<td>1,871</td>
<td>1,919</td>
<td>1,920</td>
<td>1,874</td>
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</table>

<table>
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<tr>
<th>Variables</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
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<tbody>
<tr>
<td>Housing services</td>
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<td>11,970***</td>
<td>14,872***</td>
<td>28,933***</td>
<td>10,266***</td>
<td>7,928***</td>
<td>13,153***</td>
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<td>(2,566)</td>
<td>(2,988)</td>
<td>(2,303)</td>
<td>(2,620)</td>
<td>(2,075)</td>
<td>(2,181)</td>
<td>(2,304)</td>
<td>(2,687)</td>
</tr>
<tr>
<td>(Housing services)^2</td>
<td>8,623***</td>
<td>11,789***</td>
<td>12,933***</td>
<td>14,788***</td>
<td>16,447***</td>
<td>14,809***</td>
<td>15,293***</td>
<td>16,674***</td>
</tr>
<tr>
<td></td>
<td>(419.0)</td>
<td>(504.2)</td>
<td>(415.6)</td>
<td>(530.7)</td>
<td>(417.2)</td>
<td>(393.0)</td>
<td>(455.9)</td>
<td>(467.2)</td>
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<tr>
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<td>125,195***</td>
<td>128,880***</td>
<td>141,616***</td>
<td>155,197***</td>
<td>138,547***</td>
<td>144,929***</td>
<td>140,025***</td>
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<tr>
<td></td>
<td>(3,507)</td>
<td>(3,871)</td>
<td>(2,861)</td>
<td>(2,808)</td>
<td>(2,371)</td>
<td>(2,767)</td>
<td>(2,730)</td>
<td>(3,548)</td>
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<td>1,923</td>
<td>1,900</td>
<td>1,941</td>
<td>1,929</td>
<td>1,940</td>
<td>1,967</td>
<td>1,872</td>
</tr>
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</table>
5.3 Results: the housing price function

To investigate the relationship with convexity of the house price function, we carried out a median regression using a quadratic specification of the housing price function. The estimated value of the number of housing services is the only explanatory variable in this regression. We used the estimation results of the specification that allows for time-varying neighborhood effects (specification 2 of Table 2) and compute the estimated housing services as: $\hat{q}(h) = \sum_{k=1}^{K} \beta_k h_k$.

The results of the quantile regression are given in Table 3. The coefficient of the quadratic term indicates the convexity of the housing price function. It shows a clear upward trend which is summarized in Figure 6 where we present the four year moving average of the coefficient for the quadratic term.

Although the quadratic specification allows us to investigate the key issue of convexity of the house price function in a parsimonious way, there is no particular reason why this specification should be expected to be correct. To introduce more flexibility, we have therefore also carried out local linear quantile regressions of the housing price function. Bandwidth selection is based on minimizing the mean squared error. The results are shown in Figure 7. Since the impact of the recession that started after 2007 is clearly reflected in the results, we split the figure in two panels. The first refers to the years 1995 to 2007 and clearly shows the tendency of more luxury housing to increase more in price than more decent types of housing, which results in a strong increase in the convexity of the housing price function throughout the period. The second panel shows that the convexity diminished after the year 2007, when the great recession started. Convexity decreased in 2008 and 2009, a modest recovery followed in 2010, but in 2011 the level of house prices decreased again, although least for high quality housing.

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15 See, for example, Chapter 7 of Koenker (2005).
5.4 Discussion

Our data do not enable us to study the exact relationship between house prices and income. Even if we had information about current income, this would be of limited use since economic theory suggests that the ‘permanent’ income that reflects expectations with respect to future income developments, rather than the current income, should be regarded as the proper determinant of housing demand. Since changes in consumption are related to changes in the permanent income, it is however of some interest to relate the developments in house prices to those in consumption. Figure 8 shows the annual changes in consumption volume in the Netherlands in the period 1995-2011. For the period 1995-2007 it shows positive numbers except for the year 2003. A close inspection of panel a) of Figure 7 shows that this is reflected in an exceptional downward movement of the housing price function. However, the much larger drop in consumption expenditure in 2006 is not reflected in lower – let alone less convex - house prices. The dramatic macro-economic developments in 2007/8 are reflected in the house price function, but not in consumption. However, in the years 2009-2011 there is again a close correspondence between the development of consumption expenditure and house prices.
Figure 7. Annual housing price functions 1995-2011
Another issue that should be mentioned is that the model used in this paper abstracts from housing supply by assuming a given population and a given housing stock. In reality, neither of the two were constant. It is not difficult to verify that the results of the model remain unchanged when the numbers of households and houses change by the same amounts, while \( F(y)/F(y^{\text{max}}) \) and \( G(q)/G(q^{\text{max}}) \) remain constant for all \( y \leq y^{\text{max}} \) and \( q \leq q^{\text{max}} \), respectively. The model itself suggests that this is unlikely: in a boom period prices of luxury housing will increase more than those of decent housing, which provides – all else equal – an incentive to supply more luxury housing. If this happens, the supply response to the increased convexity of the housing price function will counteract it and the long run consequences of an income shock to the housing price function can therefore be substantially different from the implications of our analysis which assumes a given distribution of housing quality. In the long run the housing price function will be determined by the supply function of houses if the total volume of demand is increasing over time. With decreasing demand the assumption of a given housing stock may also in the long run be reasonable.\(^\text{16}\)

Finally, it should be noticed that in the model we have not considered the impact of borrowing restrictions. This helped, of course, to make the case that diverging house prices could be the result of conventional market forces. However, we do not want to argue that in general such restrictions are unimportant. In the case of Amsterdam considered here, we have argued that

\(^{16}\) See Glaeser and Gyourko (2007).
the loan-to-value ratio was unimportant because of the availability of inexpensive mortgage insurance. At least until the financial crisis of 2008 banks were also willing to accept the very high loan-to-income ratios that were consistent with mortgage payments being less than – say – 30% household income at the prevailing low interest rates. Rules have been tightened somewhat since then.

If borrowing restrictions are binding, they impose an upper bound on the price households can bid. Wealth effects become more important than, since households may have the possibility to increase home equity. This will certainly be the case with binding down payment constraints as has been shown by Ortalo-Magné and Rady (2006).

6 Conclusion
This paper has proposed an explanation of the well-known phenomenon of diverging house prices by imposing a reasonable restriction on the malleability of housing that is conventionally assumed in the Muth model of housing services. Instead of a single market there is a continuum of markets for all the possible quality levels of housing. The housing price is an increasing function of the number of housing services and its curvature is determined by local supply and demand conditions. A proportional change in all incomes was shown to cause increasing convexity of this function under general conditions.

Our empirical application exploits the property of the model that the ranking of houses on the basis of price reflects the ranking on the basis of housing services. We estimate the number of housing services as a function of the housing characteristics. Using the results of this analysis, we investigated the development of the convexity of the housing price function. We found a gradual increase in the boom period 1995-2007 and a decrease followed by a weak recovery and a new decrease in the period 2008-2011.

The development over time of the convexity of the housing price function reflects that of the growth in consumption volume, which supports the idea that the development of permanent income drives that of house prices. The analysis thus suggests that the higher rate of appreciation of high-priced housing that has been observed repeatedly in various countries and time periods does not necessarily reflect the presence of a bubble or acceleration effects related to the presence of a binding down payment constraint but may well be due to the operation of the conventional forces of demand and supply in a market with substantial heterogeneity and inelastic supply.
Although the empirical analysis of this paper paid more attention to booms than busts, this should not be taken to suggest that the model is less relevant in the latter situation. To the contrary, the supply constraint is probably even more relevant in situations where house prices have decreased below construction costs than as has been emphasized by Glaeser and Gyourko, (2007).
References


