The Netherlands
Research School for
Transport, Infrastructure
and Logistics
TRAIL

11^{th} TRAIL Congress
November 2010

THE CELL TRANSMISSION MODEL WITH CAPACITY DROP

Dipl.-Inform. Thomas Schreiter, Ir. Erik-Sander Smits, Dr. Hans van Lint, Prof. Serge Hoogendoorn
Faculty of Civil Engineering, Department of Transport & Planning,
Delft University of Technology

ABSTRACT

Dynamic Traffic Management (DTA) is a growing field in transportation applications. A traffic control center, for example, needs to react quickly to incidents on a highway. It uses a traffic flow model with forecasting features, to predict the traffic state and the travel time over one hour. One of these traffic flow models is the macroscopic first-order Cell Transmission Model. In this paper, the Cell Transmission Model is expanded to reproduce the traffic phenomena of the capacity drop and of the boomerang effect by a applying a fundamental diagram in the shape of an inverted lambda.

KEYWORDS

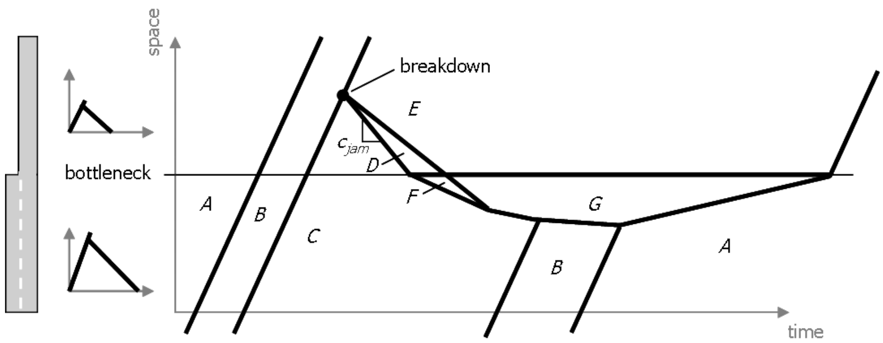
Macroscopic traffic flow model, Cell Transmission Model, capacity drop, kinematic wave theory, traffic flow phenomena

INTRODUCTION

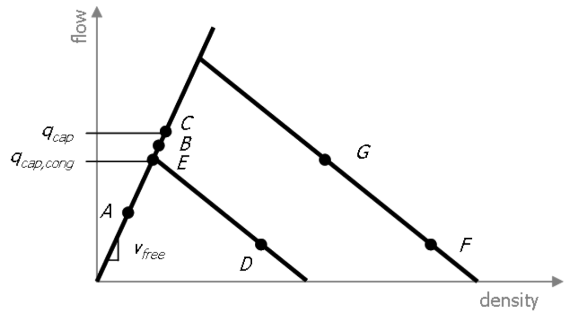
Dynamic Traffic Management is becoming important in practice nowadays as road authorities want to react to jammed highways faster and more reliably than by using simple reactive models. For this purpose, a traffic flow model is required that predicts the traffic conditions and the traffic phenomena over the time horizon of one hour. Important phenomena are, for example, the onset and dissolution of congestion and the propagation of vehicle density over space and time.

The widely-used Cell Transmission Model (CTM) by Daganzo (1994) explains these features suitably well. However, there are more phenomena that occur frequently in practice.

Figure 1 shows an example of a highway containing a bottleneck as the number of lanes reduces from two to one. The flow-density relationships are modeled by a fundamental diagram



(a) Space-Time Plane, including the road layout and the fundamental diagrams of both road parts



(b) Flow-Density Plane, the fundamental diagrams of both road parts

Figure 1: Analytical solution of a jam emergence at a lane drop by kinematic wave theory; the states D, E, F and G denote a congested traffic

(bottom). The traffic conditions over space and time are analyzed by kinematic wave theory developed Lighthill & Whitham (1955) and Richards (1956) (top).

Suppose, traffic flows into the two-lane stretch at around 2000 veh/h (state B in **Figure 1**). Under these conditions, each vehicle has a lot of space and can therefore drive at a high speed. In the one-lane section, the vehicles are forced to drive close to each other. However, it is still possible to maintain a high speed.

If the inflow increases further (C), the headways become even shorter, and it is a challenging task to keep a high speed under such a short headway. Usually, after some time, a driver chooses to slightly increase the headway. The following driver therefore has to brake, then his follower has to brake as well, and so on. Consequently, the speed drops significantly and the time headway increases (D); a breakdown occurs. This leads to congestion, with a outflow at around 1800 veh/h (E). This congestion occurred downstream of the actual bottleneck location. The congestion then moves upstream until it reaches the actual bottleneck location, where is finally remains fixed (F and G). The outflow of the bottleneck remains at around 1800 veh/h. Even if the inflow is reduced to the previous value (B), the congestion grows further. It dissolves finally, when the inflow reduces further (A).

In this example, two traffic phenomena are observed:

- The outflow during congestion (E) is lower than before the onset of congestion (B). This is called the *capacity drop* and it is frequently observed at bottlenecks.

- The congestion starts downstream of the actual bottleneck location and then moves upstream (D) to eventually stay fixed at the bottleneck location (F and G). This phenomenon is called the *boomerang effect*.

These phenomena are not captured in the basic Cell Transmission Model. Therefore, this paper shows an extension of the CTM which reproduces both the capacity drop and the boomerang effect. Due to space limitations, the methodology of one aspect of the capacity drop is shown. However, the experiments verify that both phenomena are reproduced.

THE CTM WITH THE INVERTED LAMBDA FUNDAMENTAL DIAGRAM TO MODEL THE CAPACITY DROP

In this paper, the CTM is expanded. A fundamental diagram (FD) in the shape of an inverted lambda is applied, which explicitly contains a capacity drop, namely $q_{\text{drop}} = q_{\text{cap}} - q_{\text{cap,cong}}$.

In an inverted lambda FD, unrealistic wave speeds could occur in theory, when the traffic in free flow (C in **Figure 1**) would drop to state D'. However, such wave speeds are not observed in practice. For example at the breakdown ($C \to D$), the flow drops, but the density increases. According to kinematic wave theory, the speed c_{jam} of the resulting shock wave is determined as the ratio of the flow difference and the density difference between the two states. If just the flow dropped while preserving the density, then a shock wave with infinite speed would emerge, which is unrealistic, of course. Besides, any shock wave that travels faster than v_{free} would lead to numerical instabilities in the CTM.

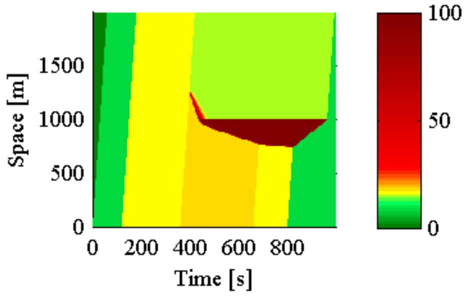
To model the jam propagation speed c_{jam} properly and to prevent such a density-conserving drop in the CTM, the supply function of the breakdown cell is modified. In the CTM, the information travel with a speed that is determined by the discretization (which is usually the free flow speed v_{free}). To ensure that the jam tail propagates in the model with c_{jam} as well, the breakdown cell must reach state D. By keeping the supply of that cell at capacity q_{cap} for the following $\alpha = \frac{v_{\text{free}}}{c_{\text{jam}}}$ time steps after the breakdown, the density increases up to that state. After these α time steps, the supply function of the usual CTM is applied again.

EXPERIMENT

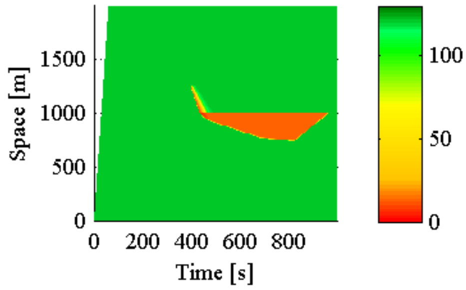
In a Matlab simulation, the proposed CTM with an inverted lambda FD is applied to the road stretch shown in **Figure 1**. The bottleneck is located at 1000 m. The discretization of space and time are 10 m and 0.3 s, respectively. The jam tail propagation speed in the bottleneck is assumed as $c_{\text{jam}} = 20\,{}^{\text{km}}/_{\text{h}}$. The free-flow speed is $v_{\text{free}} = 80\,{}^{\text{km}}/_{\text{h}}$. The inflow into the road stretch is varied.

Figure 2 presents the results as contour maps of the density, the speed and the flow. The *boomerang effect* is clearly visible, as the congestion first starts downstream of the bottleneck, and then propagates upstream to finally stay fixed at the bottleneck location. This propagation speed is significantly lower than the free flow speed.

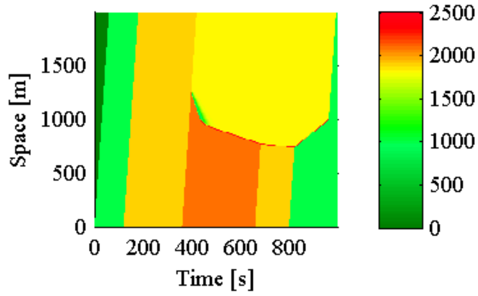
Furthermore the *capacity drop* is visible in **Figure 2(c)**. The inflow at 200 s is equal to the inflow at 700 s. However, in the first stage, the traffic flows through the bottleneck without causing congestion. In contrast, in the second stage, the discharge flow out of the bottleneck is lower, and the already existing congestion grows larger.



(a) density (in veh/km)



(b) speed (in km/h)



(c) flow (in veh/h)

Figure 2: Results of the adjusted Cell Transmission Model. The capacity drop of the bottleneck starting at 1000 m and the boomerang effect are visible.

CONCLUSIONS

In this paper, the Cell Transmission Model was modified to model two traffic phenomena. By applying an inverted lambda fundamental diagram, the capacity drop is reproduced. The modification to the supply function ensures that the jam propagation speed is realistic and that the model is numerically stable. Furthermore, the boomerang effect is reproduced as well. Since the capacity drop is an often observed phenomenon, this modified CTM with the inverted lambda FD is therefore suitable to predict highway traffic more precisely than the standard CTM. Dynamic Traffic Management is therefore expected to improve, especially at recurrent bottlenecks during peak hours.

REFERENCES

Daganzo, C. F. (1994) The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory, *Transportation Research Part B: Methodological*, 28(4), pp. 269 – 287, URL `http://www.sciencedirect.com/science/article/B6V99-466CFRH-98/2/51992b6da4383e0faf83a17023eb2f01`.

Lighthill, M., G. Whitham (1955) On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads, *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences (1934-1990)*, 229(1178), pp. 317–345.

Richards, P. (1956) Shock Waves on the Highway, *Operations research*, 4(1), pp. 42–51.